Extracting Tail Risk from High-Frequency S&P 500 Returns*

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Abstract

This paper proposes to extract tail risk from a risk-neutral mean-adjusted expected shortfall of high-frequency stock returns. Risk adjustment is based on a nonparametric estimator of the state price density that does not use option prices and relies solely on a stock index returns. This makes the measure methodology applicable to many financial markets with illiquid or nonexistent options. Empirically, the tail risk factor extracted from S&P 500 returns has a 90% correlation with the options-based VIX index and predicts well realized jumps in the stock market index at various frequencies. We document a persistent negative relation between tail risk and one-day ahead returns of several assets classes. Consistent with the crash-insurance property of put options, tail risk predicts positive one-day ahead returns for portfolios long out-of-the-money, short in-the-money put options. An analysis of equity portfolios sorted on exposure to tail risk reveals a premium for bearing such a risk, even after controlling for known and established factors related to cross-sectional variability. This cross-sectional analysis is robust to the inclusion of uncertainty indexes, as well as macroeconomic and volatility measures.

Keywords: Tail Risk, Risk-Neutral Measure, Expected Shortfall, Intra-day Market Returns.

JEL Code: G12, G13, G17.

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1 Introduction

Market participants and regulators alike value highly reliable high-frequency measures of stock market tail risk. Currently, they can rely on two prominent measures developed by Bollerslev and Todorov (2011) and Weller (2019). The first paper uses high-frequency intra-daily data and short maturity out-of-the-money options on the S&P 500 index to construct an Investors Fears index. The second paper stresses the potential limitations imposed by the rarity of liquid, deep out-of-the-money options[1] and proposes a new methodology that relies on the cross-section of bid-ask spreads. In terms of risk factors associated with extreme events, the second measure captures the aggregate economic shocks and the potential systemic threats underlying the cross-section of realized stock returns, while the first measure picks up the risk factors extracted from liquid options on the S&P 500 index. Others have used high-frequency data to compute or model tail risk based on Value-at-Risk or Expected Shortfall measures (Davies, 2016; Huggenberger et al., 2018).

In this paper, we build a new measure of tail risk by using high-frequency data, a risk-neutralization algorithm, and expected shortfall as a measure of risk. The expected shortfall is a well-known coherent measure of extreme risk often adopted by practitioners and suggested by the Basel III agreement. We compute a high-frequency version of this measure using intra-day data on stock market returns. The risk-neutralization procedure is based on a nonparametric adjustment of the raw returns. Motivated by Ait-Sahalia and Lo (1998) and Ait-Sahalia and Lo (2000), the risk adjustment puts more probability weight on extreme returns without neglecting pricing information coming from Euler equations. In terms of data, our methodology can be easily applied to different markets and assets for which intra-day returns, at any frequency, are available, avoiding the need to rely on option prices or bid-ask spreads.

To estimate the tail risk measure, we follow Almeida et al. (2017). They show that it is possible to recover risk-neutral counterparts of traditional risk measures without the need for option data. They compute a risk-neutral expected shortfall based on a cross-

[1] Few countries have a rich cross-section of index options, with a reasonably comprehensive set of liquid options. In other developed countries, the contrast with the U.S. market is remarkable. Japan and Germany, the second and third largest markets in terms of available cross-sectional index options, represent less than 30% and 20% respectively of the S&P 500 options market. In emerging countries, the index options market is either non-existent or illiquid.

section of daily portfolio or security returns[2]. In this paper, we rely solely on an index of stock returns but we use high-frequency intra-day data to enlarge the tail information. This provides a conditional tail risk measure that is based on the most recent information to isolate extreme events.

An important concern with our measure may regard its capacity to capture the forward-looking information about future states of nature contained in option prices. Recently, Bollerslev et al. (2015) proposed an option-based measure designed to capture jump tail risk[3]. We find that our measure has a correlation of about 60% with this tail risk measure. A smoothed version of our tail risk measure has a 90% correlation with the daily VIX index[4]. Figure 1 reveals that both series co-move in a large portion of the sample, providing some reassuring evidence on the adequacy of the option-less risk adjustment in our measure[5].

Given the fundamental connection between jumps and tail risk, in particular in a high-frequency environment, we follow the methodology of Weller (2019) to construct a time series of realized intra-day jumps for market returns. Using this realized measure we show not only that tail risk is significantly related to contemporaneous jumps but also that it can predict realized jumps both one-day and one-week ahead. Again, this effect persists even when we control for several other explanatory variables. We include the VIX index, a measure of realized volatility, and crash-sensitive measures (one of a macroeconomic nature – the Aruoba, Diebold and Scotti index – and one representing uncertainty –the Economic Policy Uncertainty index of Baker et al., 2016). An interesting finding is the interaction between the VIX and tail risk in relation to the realized jumps measure. While in univariate regressions each variable has a positive coefficient, the VIX

[2]Kelly and Jiang (2014) also use a large cross-section of observed returns to compute a tail risk measure by assuming that asset return tails follow a power law. With particular emphasis on the financial sector, Allen et al. (2012) and Brownlees and Engle (2017) adopted VaR and Expected Shortfall measures to estimate systemic risks. In this literature, the estimated tail risk measures are calculated on a monthly or weekly basis rather than daily.

[3]Bollerslev et al. (2015) rely on a nonparametric estimation of risk-neutral tails to perform a decomposition of the variance risk premium into diffusive and jump components. This decomposition reveals that the latter is responsible for most of the return predictability.

[4]The VIX, originally designed to measure the expected volatility of the S&P 500 index over the next 30 days, abstracts from investors sentiment, market fear or tail risk realizations. Martin (2017) shows that it measures expected risk-neutral entropy instead of expected risk-neutral variance, being more sensitive to left-tail events.

[5]Almeida et al. (2017) performed an extensive comparison between a lower-frequency (monthly) version of our tail risk measure and a corresponding option-based measure built using Ait-Sahalia and Lo's (2000) methodology and found significant similarities in their empirical properties.

changes sign when tail risk is added to the regression. Moreover, tail risk is the most important variable, both in magnitude and in explanatory or predictive power. These findings are in line with results in Bollerslev et al. (2015) who decompose the VIX into a jump tail risk component and normal-sized price fluctuations.

To further assess the empirical relevance of our tail risk measure, we conduct an extensive analysis of its relationship with several market features. First, we document a natural negative relation between current tail risk and market returns. This negative relation persists for one-day ahead predictive regressions, but mean-reverts after two days. Additionally, we also find that the market-implied tail risk is related to a flight-to-safety effect: the contemporaneous relationship with Treasury ETF returns is positive and mean-reverts after two days. We also show that our results are robust to the inclusion in the regressions of the aforementioned volatility and crash-sensitive variables as well as the FEAR factor from Da et al. (2014).

As in Bollerslev et al. (2015), we also analyze the relationship between our tail risk measure and returns from portfolios comprised of stocks sorted according to various characteristics: market capitalization, book-to-market value, momentum, market beta, and variance. Two salient features arise from this analysis. First, all one-day ahead predictive tail risk betas for the portfolios analyzed are negative and statistically significant. Nonetheless, taking a closer look at the results for industry portfolios, a significant heterogeneity among sectors is noted. In particular, financial and mining sectors suffer the biggest losses after tail risk shocks. Also, while small and big firms react similarly to tail risk shocks, high-minus-low portfolios formed on book-to-market and momentum reveal that one-day ahead returns for high book-to-market firms and loser firms have higher tail risk betas.

To understand how our nonparametric risk-neutralization is related to option returns through the tail risk measure, we construct five portfolios of put options on the S&P 500 index based on moneyness. Since these derivative contracts provide crash insurance against stock market meltdowns (Kelly and Jiang, 2014, Bollerslev and Todorov, 2011), we expect that our tail risk measure will predict higher returns for deep out-of-the-money options. We find that it is indeed the case and that there is a monotonic decay of betas across moneyness. In particular, the return of a portfolio long on deep out-of-the-money puts and short on deep in-the-money puts has a positive statistically significant beta on tail risk. We also construct portfolios sorted according to tail risk exposure and measure their alphas with respect to their returns after controlling for a three-factor Fama-French model and a four-factor Cahart model. For all holding periods (one day, one week and one month after portfolio assignment), returns are decreasing with tail risk exposure. Alphas are negative and are statistically significant for daily and weekly holding periods.

From an asset pricing theory perspective, if an asset pays in a bad state of nature, investors will require lower expected returns to keep it in their portfolios since it provides a hedge (Almeida et al., 2017, Kelly and Jiang, 2014). Our measure is positive and rises when aggregate market returns are low. Therefore, assets whose payoffs are high when our tail risk is high provide insurance against severe stock market movements. Thus, following the approach of Ang, Chen, and Xing (2006) and complementing the analysis performed in Almeida et al. (2017), we tested this hypothesis by analyzing portfolio returns sorted on exposures to our tail risk measure. As in Almeida et al. (2017), we find a positive, statistically significant, return associated with a long-short portfolio formed on tail risk exposure for all the holding periods analyzed. Most importantly, the excess returns generated by the long-short portfolios are not captured by the traditional Fama-French-Cahart factors and are robust to several double-sort analyses.

Our paper is intrinsically related to several strands of the financial literature. First, we contribute to the growing literature on the estimation of tail risk and systemic risk measures as in Kelly and Jiang (2014), Allen et al. (2012), Bollerslev et al. (2015), Siriwardane (2013), Brownlees and Engle (2017), Adrian and Brunnermeier (2014), Bali et al. (2009), among others. In contrast to these papers, we contribute to the literature by providing a methodology to compute tail risk in high-frequency environments that is virtually applicable to any set of returns.

Our paper is close in spirit to Weller (2019) who developed a real-time tail risk measure based on intra-day bid and ask quotes. In contrast, however, while Weller (2019) focuses on the natural relationship between tail risk and jumps, providing substantial evidence of jumps' prediction using intra-day data, we focus on the broader relationship between tail risk, market returns and the cross-sections of both stocks and options. Additionally, while our aim is to provide a measure that is not data intensive, Weller (2019)'s measure considers a panel of 2800 firms for its baseline one-factor model.

Aiming at capturing an "investor sentiment component" related to the VIX index, Da et al. (2014) rely on primary data from Google Trends to develop a daily sentiment index extracted from a pool of users. They document a negative relationship between their FEAR index and current market returns, and also a return-reversal feature for this index. Despite the fact that our measure is based on intra-day market data rather than investors' sentiment data, we show that the same qualitative results documented by Da et al. (2014) hold in our analysis[6]. With a similar methodology, Baker et al. (2016) extract information from newspapers to construct an Economic Policy Index (EPU). They find that their constructed index is positively associated with higher volatility, and expand results in Bloom (2009) in terms of macroeconomic forecasting ability. Our predictability results are robust to the inclusion of Baker et al. (2016)'s measure as a control.

Finally, our paper also expands the analysis in Bollerslev et al. (2016). Bollerslev et al. (2016) consider an extension of the CAPM model in which a beta from a jump component is added to the market beta. They found that the only significant premium is associated with the jump component. In contrast to that paper, we offer a new tail risk measure that is not dependent on options data and does not rely on any parametric dynamics for the market return.

The rest of the paper is organized as follows. Section 2 describes how we construct a risk-neutralized shortfall measure of tail risk based on the market returns, how we estimate the measure and how the measure relates to other measures proposed for tail risk. Section 3 relates our tail risk measure to realized jumps in the stock market index at various frequencies, controlling for VIX, realized volatility and other crash-sensitive measures. Section 4 analyzes the contemporaneous and predictive relationships between tail risk, the aggregate market returns and several asset classes. Section 5 investigates the implications of tail risk for option portfolios. Section 6 presents results of a risk premium analysis using a cross-section of stock returns and various sorting characteristics. Finally, section 8 concludes with a summary of the main results.

[6]Our predictive results are also robust to the inclusion of their FEAR measure when considering the sample where both tail risk measures overlap.

2 A Nonparametric Tail Risk Measure

2.1 Definition

In this paper, we aim at building a simple tail risk measure that depends solely on aggregate stock returns observed at high frequency. We use intra-daily returns on the S&P 500 index to estimate a daily tail risk measure by following the methodology developed in Almeida et al. (2017), who compute a monthly tail risk measure from a cross-section of portfolio returns. They use a risk-neutral, mean-adjusted, expected shortfall (ES), a coherent risk measure (Artzner et al., 1999) that overcomes the main deficiencies of the Value-at-Risk (VaR)[7]. We define the market excess expected shortfall as follows:

$$TR_t = E^{\mathcal{Q}_t(R)}[(VaR_\alpha(R_\tau) - R_\tau)^+] \tag{1}$$

where t is the day for which we are calculating the tail risk, τ denotes the possible states of nature (defined as the intra-day, realized index returns), α is the VaR threshold and $\mathcal{Q}_t(R)$ indicates the risk-neutral density as a function of the high-frequency returns at day t. Note that in this design we model $\mathcal{Q}$ as a function of the observable returns. Since we compute the tail risk at daily frequency using the intra-day return realizations, there is no overlapping of data between days. Therefore the measure quickly incorporates the market tail information.

2.2 Risk Neutralization

To risk-neutralize the returns, we follow the option-free nonparametric approach introduced in Almeida et al. (2017). It relies on the generalization of Hansen and Jagannathan (1991) proposed by Almeida and Garcia (2017) to estimate a stochastic discount factor that captures the higher moments of the basis asset returns. While Hansen and Jagannathan (1991) minimize a quadratic loss function given a set of basis assets, Almeida and Garcia (2017) minimize a family of convex functions (so-called Cressie-Read family) defined in the space of admissible and strictly positive SDFs. These convex functions measure the distance between an admissible SDF and the constant SDF of a risk-neutral

[7]Being a point-wise measure, VaR ignores the behavior of the tail of the distribution beyond its threshold. Also, due to the relative scarcity of tail events in historical time series, VaR modeling is challenging and can lead to overestimation of risks in calm periods (Berkowitz and OBrien, 2002) and underestimation during crisis (Jorion, 2019).

economy[8].

Given the extensive analysis of the properties of the nonparametric estimator in Almeida et al. (2017) and Almeida and Garcia (2017), we provide here only a brief description of the methodology that we followed to calculate our benchmark measure. For the sake of simplicity, we specialize the problem of SDF estimation to the Hellinger estimator, a particular case of the general Cressie-Read family approach of Almeida and Garcia (2017)[9].

Let $(\Omega, \mathcal{F}, P)$ be a probability space, and R denote a K-dimensional random vector on this space representing the returns of K primitive basis assets. In this static setting, an admissible SDF is a random variable m for which $E(mR)$ is finite and satisfies the Euler equation:

$$E(mR) = 1_K, \tag{2}$$

where 1_K represents a K-dimensional vector of ones.

For a sequence of (m_τ, R_τ) that satisfy Equation (2) for all t, and observing a time series $\{R_\tau\}_{\tau=1,\dots,T}$ of basis assets returns, we assume that the composite process (m_τ, R_τ) is sufficiently regular so that a time series version of the law of large numbers applies[10]. Therefore, sample moments formed by finite records of measurable functions of data R_τ will converge to population counterparts as the sample size T becomes large.

Given a sample of basis assets returns, the set of admissible SDFs will depend on the market structure. The usual case, considering a static one-period model, is to have an incomplete market, i.e., the number of states of nature (T) larger than the number of basis assets K. In such a case, an infinity of admissible SDFs will exist, and if there is no in-sample arbitrage on the basis assets payoff space (Gospodinov et al., 2016), there will exist at least one strictly positive SDF (see Duffie, 2001). For each strictly positive SDF there will be a corresponding risk neutral density. The fundamental difference between this paper and Almeida et al. (2017) is that, instead of using daily realizations of R_τ as the states of nature, we rely on intra-day realizations of the market returns. By doing so,

[8]Assuming a constant short-term rate and homogeneous physical probabilities, just as in a VaR historical simulation, we can obtain a direct correspondence between SDFs and RNDs.

[9]The choice of this particular estimator in the family is explained in detail in Almeida et al. (2017).

[10]For instance, stationarity and ergodicity of the process (m_t, R_t) are sufficient (see Hansen and Richards, 1987). Also, we further assume that all moments of returns R are finite in order to deal with general entropic measures of distance between pairs of stochastic discount factors.

we can increase the number of states T by simply increasing the frequency of the data used (say, from 1 hour to 15 minutes). Additionally, for tail risk estimation purposes, we are able to compute an aggregate market tail risk directly from the observed returns. From a factor model perspective, with the market return being the only source of risk, our non-parametric SDF approach resembles the CAPM. However, in contrast to the CAPM, our methodology generates a stochastic discount factor that is non-linear in the market returns, incorporating its higher-order moment information.

Given the Hellinger discrepancy function $\phi(m) = -4(m^{1/2} - a^{1/2})$ the generalized, in-sample, minimum-discrepancy problem proposed by Almeida and Garcia (2017) can be stated as:

$$\begin{aligned} \hat{m}_{MD} = \underset{\{m_1,\dots,m_T\}}{\arg\min} \quad & \frac{1}{T}\sum_{i=1}^{T}\phi(m_i) \\ \text{subject to} \quad & \frac{1}{T}\sum_{i=1}^{T} m_i(R_i - \frac{1}{a}1_K)] = 0_K \\ & \frac{1}{T}\sum_{i=1}^{T} m_i = a \\ & m_i \geq 0 (\text{or } m_i > 0)\ \forall\, i \end{aligned} \tag{3}$$

In this optimization problem, restrictions to the space of admissible SDFs come directly from the discrepancy function ϕ. The conditions $E(m\left(R - \frac{1}{a}1_K\right)) = 0_K$ and $E(m) = a$ must be obeyed by any admissible SDF m with mean a. In addition, whenever there is a strictly positive solution the implied minimum discrepancy SDF is compatible with absence of arbitrage in an extended economy that considers derivatives over the underlying basis assets[11]. The choice to impose a non-negativity or strict positivity constraint in the optimization problem is dictated by the choice of the discrepancy function $\phi(.)$ (see Almeida and Garcia (2017) for a detailed analysis).

Despite the straightforward interpretation of the problem in (3), its solution is not easy given that the number of unknowns is as large as the number of observations in the sample. Therefore, Almeida and Garcia (2017) show that one can solve an analogous simpler dual problem:

[11] It is important to note that the homogeneous probability assumption will not affect the key insights we derive from this methodology and, if desired, one could also consider a kernel density to model the physical probabilities without additional complications.

$$\hat{\lambda} = \underset{\alpha \in \mathbb{R},\, \lambda \in \Lambda}{\arg\sup}\ a * \alpha + \frac{1}{T}\sum_{i=1}^{T} \phi^{*,+}(\alpha + \lambda'(R_i - \frac{1}{a}1_K)) \tag{4}$$

where $\Lambda \subseteq \mathbb{R}^K$ and $\phi^{*,+}$ denote the convex conjugate of ϕ restricted to the non-negative (or strictly positive) real line.

$$\phi^{*,+} = \underset{w \in [0,\infty) \cap \text{domain } \phi}{sup} \quad zw - \phi(w) \tag{5}$$

In this dual problem λ can be interpreted as a vector of K Lagrange multipliers that come from the Euler equations for the primitive basis assets in (3). For the specific case of the Hellinger estimation, closed-form formulas are obtained for λ and $\hat{m}_{MD}$:

$$\hat{\lambda}_H = \underset{\lambda \in \Lambda_{CR}}{\arg\sup}\ \frac{1}{T}\sum_{i=1}^{T} \left(2a^{1/2} - 2\left(a^{1/2} - \frac{1}{2}\lambda'\left(R_i - \frac{1}{a}1_K \right) \right)^{-1} \right), \tag{6}$$

and:

$$\hat{m}^i_{MD} = a \frac{\left(a^{1/2} - \frac{1}{2}\hat{\lambda}'_H \left(R_i - \frac{1}{a}1_K \right) \right)^{-2}}{\frac{1}{T}\sum_{i=1}^{T} \left(a^{-1/2} - \frac{1}{2}\hat{\lambda}'_H \left(R_i - \frac{1}{a}1_K \right) \right)^{-2}}. \tag{7}$$

To obtain the risk neutral probabilities associated with each observation interpreted as a state of nature, we distort the usual $1/T$ measure by the computed SDF in (7) adjusted by the interest rate[12]:

$$\pi_i^{RN} = \frac{m_i(1+r)}{T}. \tag{8}$$

To understand the effect of the estimated risk neutral density on our tail risk measure, we derive a Taylor expansion of the expected value of $\phi(m) = -4(m^{1/2} - a^{1/2})$ around the SDF mean a. Noting that $\phi(a) = 0$, $\phi'(m) = -2m^{1/2}$, $\phi''(m) = m^{3/2}$, $\phi'''(m) = (-3/2)m^{5/2}$, and $\phi''''(m) = (-3/2)(-5/2)m^{-7/2}$, Taylor expanding ϕ and taking expectations on both sides we obtain:

[12] This framework is similar to the entropic estimator for the SDF proposed by Stutzer (1996) but with a different minimizing function. In fact Almeida and Garcia (2017) showed that the minimum entropy estimator is a special case of the Cressie-Read estimators.

$$E(\phi(m)) = \frac{a^{-3/2}}{2}E(m-a)^2 + \frac{-(3/2)a^{-5/2}}{3!}E(m-a)^3 + \frac{(15/4)a^{-7/2}}{4!}E(m-a)^4 + ... \quad (9)$$

Analyzing this Taylor expansion two important aspects regarding the weights attributed to skewness and kurtosis are notable. First, differently from the Hansen and Jagannathan (1991) approach, the Hellinger estimator is dependent on higher-order moments of the underlying returns. In particular, we see that the absolute weight given to kurtosis is smaller than the one given to skewness. Also, besides the differences in their magnitude, the weights given to skewness are negative, while the weights assigned to kurtosis are positive. In other words, the state price density will be higher when kurtosis is higher or when skewness is lower, therefore revealing the skewness "preference" and kurtosis "aversion" characteristics of the implied estimator (which are in line with findings in Kraus and Litzenberger (1976) and Backus et al. (2011)).

2.3 Estimation

To construct the baseline measure, we use 15-minute intra-day data for the open-to-close returns on the S&P 500 index, extracted from Bloomberg, from 01/02/2008 to 01/07/2015, as in Andersen et al. (2019). Our sample starts at 09:30 a.m. and ends at 4 p.m. every day. We set the VaR threshold at $\alpha = 20\%$ so we can capture the daily dynamics of the tails of the returns distribution. The choice of the 15-minute time interval provides a compromise between reducing microstructure noise and gathering enough information to observe jumps[13]. With an intra-daily 15-minute interval, we obtain $T = 26$ observations per day. Since the risk-neutralization of returns changes the probabilities with respect to the $\frac{1}{T}$ equal weights, the number of effective observations per day used to estimate our tail risk measure will vary from day-to-day, rarely exceeding five observations. The top panel of Figure 1 plots the daily estimated measure from 01/02/2008 to 01/07/2015 together with the daily VIX index. It illustrates the ability of the tail risk measure to capture extreme stock market events as well as the correlation with economic

[13]One could argue that the use of higher-frequency data, say at one-minute intervals, might translate into a more precise measure. We believe, in line with Andersen et al. (2019), that expanding the frequency of the data might exacerbate market-microstructure related problems in the estimated measure. For the sorting portfolio procedures we re-estimate our measure using one-minute data and provide some evidence that the overall results hold.

conditions on a daily basis, corroborating the findings in Almeida et al. (2017). Notably, our measure is very volatile and features various peaks, often coinciding with periods of high expected volatility. Not surprisingly, the measure captures both the financial crisis and the European debt crisis, peaking in both of these events.

The middle panel of Figure 1 plots our tail risk measure and a daily tail risk time series estimated from higher-frequency tail risk series computed by Weller (2019)[14]. The two series feature a strong co-movement especially in periods of market turmoil, which shows that the daily tail risk measure of Weller aggregates well the cross-sectional information to capture market events.

In the bottom panel of Figure 1, we plot three series: a 10-day moving average of our daily tail risk measure, a 10-day moving average of the Weller daily taik risk series, and the daily VIX. The co-movements between these series are even more striking. To put these strong relations into perspective, let us recall that we estimate our tail risk using intra-day returns on the S&P 500 index that we risk-neutralize with a nonparametric procedure. In contrast, the VIX index is based on the expected volatility extracted from the whole cross-section of short-maturity option prices on the stock index. Also, while the VIX measures the expected market volatility, our measure is specifically designed to capture extreme movements in the stock market returns. With respect to the Weller measure, we use much less information than the high frequency measures of bid-ask spreads of a large cross-section of equity securities. Therefore, given both their methodological and theoretical differences, it is somewhat surprising that these measures align so well.

In Table 1, we report the correlations between the estimated Hellinger Tail Risk, its physical counterpart and a moving average of the Hellinger Tail risk with several crash-sensitive measures. As it is evident from Figure 1 the correlations with the Weller tail risk measures are high: 0.64 between the daily measures and 0.84 between the 10-day moving averages. They are also high with the VIX: 65% for the Hellinger Tail risk and 90% for the moving average. This high correlation is also noted for Bollerslev et al. (2015) fear measure, another option-based tail measure. Interestingly, the Hellinger moving average carries a lower correlation with realized volatility, which is based on the objective and not a risk-adjusted measure. Also, the correlations between the expected shortfall and all risk-adjusted measures of tail risk are substantially lower than with our Hellinger tail-risk

[14]The original series built by Weller (2019) are based on 15-minute and hourly intra-daily intervals. We construct a daily series by averaging each day the hourly measures.

measures. This indicates that the nonparametric risk adjustment provides a correction to the objective measure in line with the other risk-adjusted measures based on options or the cross-section of bid-ask spreads.

While it is expected for tail measures to co-vary together, we also compute the correlations of the Hellinger Tail Risk with indexes based on uncertainty and macroeconomic variables. Table 1 reveals that our measure is positively correlated with uncertainty, measured as the Economic Policy Uncertainty (EPU) index from Baker et al. (2016) and negatively correlated to the Aruoba, Diebold and Scotti (ADS) index, a pro-cyclical business-conditions index from the Philadelphia Federal Reserve Bank. While similar in essence to the EPU index the correlation of our Hellinger Tail Risk with the FEAR index of Da et al. (2014) is almost negligible. Not surprisingly, the correlation between our measure and an equity-based EPU is also positive and higher than the correlation with the EPU itself.

Finally, we note that the estimated tail risk measure is positively correlated with several other crash sensitive market measures: Emerging Markets spread with the spot Treasury curve from BofA, the Stock Market Crashes measure by the Cleveland FRB as the ratio of the current value for the S&P 500 index and its maximum over the last 365 days, the contribution of the banking sector to overall stock market volatility from the Cleveland FRB, the spread between Moody's Seasoned Baa Corporate Bond and 10-Year Treasury Constant Maturity and the noise measure of Hu et al. (2013).

3 Tail Risk and Stock Market Jumps

Taken together, the heuristic results of the previous section provide some preliminary evidence that our daily tail risk measure captures large fluctuations in market risk. In this section, we formally establish the relations between tail risk and stock market jumps. Our methodology follows closely Weller (2019) approach using a daily framework instead of intra-day data. We measure aggregate stock market jumps as intra-day deviations from the historical, intra-day, mean for price increments. More specifically, let $\Delta P(\tau-1,\tau)$ denote the price variation between $\tau-1$ and τ withing each intra-day price interval. We construct five preliminary downside jump measures as follows: $DJ_t(i) = \sum_{\tau=2}^{T} I[\mu_t - (i+1)\sigma_t < \Delta P(\tau-1,\tau) < \mu_t - i\sigma_t]$ for $i = 1, \ldots, 4$ and $DJ_t(5) = \sum_{\tau=2}^{T} I[\Delta P(\tau-1,\tau) < \mu_t -$

$5\sigma_t]$ where μ_t and σ_t are the respective mean and standard deviation of price increments for the preceding 50 days. The aggregate jump measure is a weighted average of the five preliminary measures, as in Weller (2019), as follows:

$$\text{D. Jump}_t = \frac{\sum_{i=1}^{5} iDJ_t(i)}{\sum_{i=1}^{5} i}. \tag{10}$$

By letting both the mean and the standard deviation of the price increments vary over time, we allow our jump measure to capture jumps conditional on volatility. This also avoids using future data on price increments when constructing our realized jump measures or setting ad-hoc thresholds for the jumps calculation.

Our key analysis in this section is a regression that takes the following form:

$$\text{D. Jump}_{t+h} = \alpha + \beta TR_t + \sum_{k=1}^{K} \gamma_k X_k + \epsilon_{t+h} \tag{11}$$

where TR denotes the Hellinger Tail Risk measure and X includes additional explanatory variables. It should be noted that all variables are standardized (0 mean and unit variance).

First, we set $h = 0$ and therefore look at the contemporaneous relation between tail risk and jumps. Table 2 shows the results of this analysis for the aggregate jump measure. In Panel A we present the estimated coefficients for univariate regressions. Except for the ADS index, a business-conditions measure based on macroeconomic variables, all explanatory variables have positive, statistically significant, betas. Among the selected set of explanatory variables, we note that the β with the highest magnitude is the one associated with tail risk, being almost 60% higher than the one for the realized variance, the second largest coefficient. More strikingly, while most of the explanatory variables have relatively small estimated R^2, the value associated with tail risk is as high as 16%, more than double the one for the realized variance measure, which is the second largest.

In Panel B of Table 2 we report the results for multivariate regressions where we add one explanatory variable at a time. Four features can be noted. First, for all four regressions, the coefficient on tail risk remains stable when compared to the univariate regression. Second, while for the univariate regression the coefficient on the VIX index was positive and statistically significant, it becomes negative and remains significant when we control for tail risk. Therefore, once we control for negative tail realizations, an increase in future variance captured by the VIX should lower the number of negative jumps and have a positive effect on returns. Third, the estimated coefficient for realized variance is reduced by half when we control for tail risk and is not significant anymore, while the EPU variable does not have any additional explanatory power. Finally, the estimated R^2 are only slightly higher when we add the additional explanatory variables when compared to the univariate framework.

Table 3 further investigates these relations by running similar multivariate regressions for the five components of the aggregate jump measure. Overall, we note that the main features persist for each component, with the tail risk being the main driver of the jump realizations. For each component, the univariate R^2 from tail risk regressions is also high when compared to the multivariate framework, representing on average more than 75% of the later. Also, while the tail risk beta is monotonic, being higher for smaller jumps, all the estimated betas are statistically significant. In fact, if one takes into account that the average for the $DJ_t(1)$ is approximately 2, meaning that for each day we observe on average two "jumps" between 1 and 2 standard deviations from the last 50-day mean, and the $DJ_t(5)$ average is around 0.07, the estimated coefficients for the tail risk convey additional meaning. That is, an increase of one standard deviation in current tail risk is associated with 0.26 more $DJ_t(1)$ on average, about 13% of the mean value. The same one standard deviation rise in tail risk is associated with an increase of 0.10 $DJ_t(5)$ on average, about a 100% increase relative to its average.

In Table 4, we present the results for one-day ahead predictive regressions ($h = 1$). Similar to the contemporaneous setup, univariate regressions reveal that the most relevant variables in terms of prediction are the tail risk and the realized variance. We must stress however that the relation between realized variance and future jumps is not surprising given the rolling window methodology to calculate the jump measure. By construction, increases in volatility imply harsher jump thresholds in the future. Still, the coefficient on realized volatility is positive and of the same magnitude as for the tail risk in the predictive regressions. When comparing the obtained results with the contemporaneous regressions, we also find closely related patterns. In a multivariate setting, the tail risk coefficient is positive and statistically significant for all regressions, with the magnitude being almost unchanged when comparing to the univariate counterpart. As expected, while in contemporaneous regressions a one-standard deviation increase in tail risk was associated with a 0.11 average increase in the downside jump measure, the magnitude in the predictive regressions reduces to 0.02. Contrary to the contemporaneous regression, the VIX is not significant by itself, but it plays a significant role when realized volatility enters the regression. Both the EPU and the ADS do not add any predictive power.

When we consider the results in Table 5 for the segregated jump predictive regressions the picture is slightly different. With one exception, the $DJ_t(4)$ component, the tail risk beta is at least 25% higher than the one for the realized variance. Also, out of five components, three of the estimated betas for the realized variance are not significant, while all of them are for the tail risk (at a 10% significance level). The patterns for VIX, ADS and EPU are also stable, being for the most part irrelevant in statistical terms. Table 5 also reveals that for the jump measure components the R^2 of univariate regressions where the sole explanatory variable is the tail risk are relatively high, ranging from 20% to 50% when compared to the multivariate R^2.

Finally, in Table 6 we present the results for weekly jump predictions. To avoid overlapping data, in the first day of each week, we compute all the measures used as explanatory variables. Then, we aggregate each jump component from the second day of each week to the first day of the next week. Results for these regressions revealed that, as before, the R^2 and the betas for the tail risk maintain their patterns. Interestingly, the realized volatility loses its explanatory power while the VIX, a forward-looking measure, becomes statistically significant.

Overall, this section confirms the suggestive evidence provided by correlations appearing in Table 1 and Figure 1. Our findings are also comparable to Weller (2019) although we only work with daily jumps while his analysis can be extended to both daily and intra-day settings. More importantly, we verify that our measure is intimately related to realized jumps in ways not spanned by volatility.

4 Tail Risk and Asset Returns

Another important aspect of the empirical application is to verify the relationship between our tail risk measure and both contemporaneous and future returns among different asset classes as in Bollerslev et al. (2015) and Da et al. (2014). We start by analyzing the contemporaneous and predictive properties of tail risk with respect to market returns via regression analysis. Then, we expand this framework to different asset classes and portfolios returns.

In the regression framework we consider a number of control variables. We adopt a measure of realized volatility based on the sum of squared intraday returns following Andersen et al. (2019). For most regressions we also use the CBOE VIX index as an additional control given the forward-looking information embedded in the measure. In addition, to control for macroeconomic business conditions we use the Aruoba, Diebold and Scotti index from the Philadelphia FRB, and to control for economic uncertainty we include the EPU index of Baker et al. (2016).

4.1 Tail Risk and Market Returns

The tail risk literature has documented the existence of a long-term premium for bearing disaster risk (Almeida et al., 2017; Kelly and Jiang, 2014; Bollerslev et al., 2015). While at lower frequencies the risk-return relation is relatively well understood, there is no clear evidence on the high-frequency relation between market returns and tail risk. Our daily measure of tail risk built from intra-day returns will provide some evidence on this short-run relation.

We analyze the contemporaneous relation between tail risk and returns as well as the predictive ability of the tail risk measure. We rely on the following regression, considering the same control variables as before:

$$R_{t+h} = \alpha + \beta TR + \sum_{k=1}^{K} \gamma_k X_k + \epsilon_{t+h}. \tag{12}$$

Table 7 presents the results for this analysis with $h = 0, 1, 2$. Not surprisingly, both tail risk and the VIX index are associated with negative contemporaneous returns for the S&P 500 index. Also, in contrast with the jump regressions, both EPU and ADS coefficients are statistically significant for returns regressions. Interestingly, among all the explanatory variables, the biggest coefficient in magnitude is the tail risk beta[15]. Such beta indicates that a one-standard deviation increase in tail risk is associated on average with a -0.51% drop in the S&P 500. In addition, the R^2 for a univariate contemporaneous regression with the tail risk measure as a sole explanatory variable represents 66% of the

[15]Note that since all variables are standardized, these betas measure standard deviations from the mean.

total R^2.

In the predictive setting with $h = 1$ (a one-day ahead prediction), the sign of the coefficient for the VIX mean is inverted while its magnitude stays the same as for $h = 0$. However, the coefficient for tail risk remains negative and statistically significant, showing persistence in the negative effect for future one-day ahead market returns. For the two-day ahead returns ($h = 2$), the effects of tail risk partially mean revert. The realized variance and EPU have a positive statistically significant coefficient for contemporaneous predictions but their effects are not significant in predictive regressions.

The results presented in this section can be compared to those in Da et al. (2014), Bollerslev et al. (2015), and Almeida et al. (2017). While Da et al. (2014) find a one-day mean-reversion period using their FEAR measure based on internet searches, we find that shocks in our tail risk measure take a bit longer to mean revert. Using monthly data, Bollerslev et al. (2015) and Almeida et al. (2017) find a positive relationship between tail risk and future returns, a feature that is consistent with the usually expected risk-return structure in traditional asset pricing models. In contrast, we observe that this feature should be relative to the frequency of data used[16]. Our high-frequency analysis highlights some persistence in the negative responses of returns to tail risk shocks before the above-mentioned positive relation between tail risk and future returns reappears.

4.2 Tail Risk and Portfolio Returns

Bollerslev et al. (2015) provide evidence that different assets and portfolios have different exposures to the jump tail risk component of the variance risk premium and that this jump component adds to the predictability of their returns[17]. To investigate potential heterogeneous reactions to our tail risk measure, we select several assets and portfolios as the left-hand side variable in the contemporaneous and predictive regressions (12).

We first consider returns on a "safe" investment, the iShares U.S. Treasuries ETF, a daily traded fund invested in Treasuries, to analyze the "fly-to-safety" aspect on dates where tail risk is high. We select a traded ETF instead of returns on fixed maturity bonds to avoid using fitted data[18]. Arguably, Treasuries provide a safe haven in times

[16] Bandi et al. (2019) provide a formal view on the behavior of predictability as a function of the frequency of observed data.

[17] Da et al. (2014) also present evidence on the distinct reaction of various assets to increases in their FEAR measure.

[18] Fixed maturity bonds are usually built based on methodologies that interpolate observable bond

of economic distress. Therefore, the ETF allows us to analyze the potential increase in market safety demand due to shocks on tail risk. Table 8 presents results considering the same set of regressions estimated using market returns before.

As in Da et al. (2014), we find a positive "fly-to-safety" effect associated with contemporaneous increases in tail risk. This is also true for some explanatory variables considered in the multivariate setting including VIX and ADS but not for the realized variance and EPU. In the contemporaneous regressions, the betas for all variables are statistically significant but have different signs. Again, the tail-risk beta has the largest magnitude, implying a 0.12% rise on Bond ETF's returns associated with a one- standard deviation positive shock in tail risk. The mean-reversion patterns are clearly different from the regressions with market returns. While tail risk still mean reverts, the one-day ahead coefficient is close to zero. VIX does not present any mean-reversion pattern suggesting that a positive shock to VIX implies a more persistent "fly-to-safety" movement than our tail risk measure would suggest. Realized variance, ADS and EPU do not seem to be related to future Treasury returns.

We now perform a similar analysis on portfolios sorted by market beta, variance, size, book to market, and momentum. Table 9 shows the results for contemporaneous as well as predictive regressions for the market beta[19]. Given the direct connection between market betas and market returns, we expect that portfolios with higher market beta will be more exposed to our tail risk measure, which is extracted from market data. Indeed, columns 1-10 of Table 9 reveal that these portfolios have both contemporaneous and predictive tail risk betas that are negative and higher in magnitude than portfolios less exposed to the market return. For instance, while the portfolio with highest market beta has an average drop of -0.44%, one day after a tail risk shock, the one with lowest market beta has an average drop of -0.11%. Low beta stocks have close to zero correlation with contemporaneous tail risk revealing that not only these stocks are linearly orthogonal to market returns but also to large drops on market returns.

We observe a monotonic relationship between exposures to market beta and corresponding estimated betas for the tail risk. This monotonic pattern is also true between exposures to market beta and betas estimated for all control variables. For the one-day ahead predictions, the coefficients for the tail risk beta are negative, statistically

prices/returns.

[19]The market beta of a portfolio is defined as the linear exposure of this portfolio to the market return.

significant, and the biggest in magnitude when compared to the ones from the control variables[20]. Excess return portfolios formed buying low market beta portfolios and selling high market beta portfolios have a statistically positive relationship with tail risk, therefore being capable of partially hedging such a risk.

Differently from the results for the market beta portfolios, Table 10 reveals that there is no clear pattern for the estimated coefficients for the tail risk in the variance sorted portfolios. While the low minus high variance portfolio still presents a positive statistically significant contemporaneous beta with tail risk, the statistical significance is only marginal in the predictive regressions. We also do not find a monotonic relationship between the variance portfolios and tail risk exposure. Overall, the estimated contemporaneous beta ranges from -0.08 for the lower variance portfolio to -0.58 for intermediary portfolios.

With respect to size sorted portfolios, a clear distinction emerges[21]. While we still find a negative relationship between future returns and current tail risk, there is no clear distinction between small and big firms. The estimated coefficients are of the same magnitude, around -0.30, for both portfolios, with analogous magnitude to the estimated tail risk beta in S&P 500 index regressions. VIX, and ADS also present coefficients of similar magnitudes for small and big portfolios. We do find however that big firms react more to rises in current realized variance, although individually the coefficients are not statistically significant (for the small and big portfolios). The results for high and low book-to-market portfolios are also similar to the size case. In addition, there is no important distinction between them and those obtained for the market betas and the S&P 500 index regressions.

Momentum portfolios, on the other hand, have a different structure. While the magnitude of the tail risk betas with respect to momentum portfolios is still similar to that of the S&P 500 index regressions, ranging from -0.22 to -0.39, the portfolio formed with loser stocks is consistently more exposed to tail risk. This implies winners minus losers (WML) portfolios having statistically significant exposures to tail risk, in contrast to the non-significant exposures obtained by the SMB and HML portfolios above. In particular, the WML portfolios formed with small (big) firms have a statistically significant beta of 0.09 (0.17) with respect to a one-day lagged tail risk.

[20] VIX is the only control variable that presents comparable coefficients.

[21] For the sake of brevity, we drop the contemporaneous returns regressions in the upcoming analysis.

As a final analysis of the sensitivity of potentially diverse portfolio returns to our tail risk measure, we select the 49 industry portfolios available in the Kenneth French library to perform similar predictive regressions to the ones above. While the previous analysis focuses on portfolios formed by a fixed firm characteristic, industry portfolios allow us to better understand how different sectors of the economy may react to sharp declines in market returns measured by our tail risk measure.

Figure 2 plots the results for each of the 49 portfolios. In black, we present the estimated tail risk beta, in red the 10% confidence interval, in green the ADS beta and in purple the VIX beta. Although not included in the picture we also control for the EPU index and the realized variance in these regressions. The picture reveals a significant heterogeneity in the one-day ahead betas for different sectors. The Beer sector has the smallest beta in magnitude (-0.11) and the Coal industry the biggest one (-0.53). The financial sector (banks, insurance and real state financial firms) along with the mining sector (gold, mines, coal and oil) are among the ones most exposed to tail risk shocks.

5 Crash Insurance: Tail Risk and Options

Since out-of-the-money puts provide a natural hedging force against tail risk, we expect that put returns should be intimately linked to our tail risk measure. To empirically test such a link, we collect daily put options prices (bids and asks) from OptionMetrics IvyDB US database. Relying on the standards in this literature, we calculate option prices as averages between bid and ask closing values and use this data to construct daily portfolios of put index options sorted by moneyness (K/S)[22]. The portfolios are rebalanced on a daily basis and the returns are calculated based on a buy-and-hold strategy. We build five portfolios according to the moneyness of options with maturities between 1 and 45 days: deep out-of-the-money (DTOM), out-of-the-money (OTM), at-the-money (ATM), in-the-money (ITM), and deep in-the-money (DITM)[23]. We focus on short-term options since they are more likely to react to short-term movements of the underlying index.

As in previous sections, we control for the VIX index, realized variance, ADS index,

[22] where K is the strike price and S the spot price of the underlying asset.

[23] The moneyness thresholds are defined as follows: $0.9 \leq DOTM < 0.94$, $0.94 \leq OTM < 0.97$, $0.97 \leq ATM < 1.03$, $1.03 \leq ITM < 1.06$, $1.06 \leq DITM < 1.10$.

EPU index and also for our aggregate downside jump measure. Adding this jump measure allows us to partially distinguish the effect of tail risk from that coming from a pure jump component.

Panel A of Table 12 reports the results for contemporaneous regressions. In line with our intuition, given the implicit insurance provided by OTM puts, we find a positive monotonic relationship between tail risk and moneyness. In particular, while the estimated beta for the DOTM portfolios is 0.09, it is less than half (0.04) for the DITM portfolios. The estimated beta for a portfolio long on DOTM options and short on DITM options is positive and statistically significant. Panel A also presents the values for the R^2 statistics for both the multivariate setting and a univariate setting where the unique explanatory variable is our Hellinger tail risk measure. By itself, the tail risk measure explains, on average, 9% of the variation in these portfolios, that is close to 40% of the overall R^2 obtained in the multivariate setting. This indicates the intimate contemporaneous relation between tail risk and put option returns, even though the measure does not rely on option prices.

Panel B of Table 12 complements the preceding analysis with results for one-day ahead predictive regressions. The first striking result is that the persistence of options returns to shocks in tail risk is clearly stronger when compared to the previous analysis performed with market returns. Indeed, the ratio between predictive and contemporaneous betas is always greater than 75% while it achieved 60% in the market returns case. It is even higher that 100% for the DOTM portfolio (from 0.09 to 0.10). Of course, the estimated R^2s for the univariate predictive regressions are lower than for the contemporaneous regressions but they remain high (ranging from 1% to 3%) when compared to the ones for the multivariate regressions (ranging from 3% to 6%).

We also analyze how traditional factor models' alphas are related to the tail risk exposures of estimated put option portfolios. Panel C reports the estimated alphas for the Fama and French three-factor and the Fama-French-Cahart four-factor models. As documented in Kelly and Jiang (2014), we find a negative statistically significant alpha for the DOTM portfolio. Thus, selling OTM puts produces on average positive returns that cannot be explained by traditional factor models. The last column of this panel presents the correlation between the vector of estimated beta exposures of each option portfolio to our tail risk measure and the vector of alphas obtained for each option portfolio (for both factor models). Irrespective of the set of factors considered, we find a 72% negative correlation between the vector of tail risk betas and the vector of portfolios' alphas, revealing a strong relationship between abnormal returns and tail risk exposures.

6 Tail Risk Pricing

Up to this point our analysis focused on contemporaneous and predictive relationships between tail risk and a set of investment returns. In this section we want to go beyond correlations and to start thinking about risk premia. From an asset pricing perspective, assets with high payoffs in bad states of nature represent good hedges. Therefore investors require lower expected returns to hold such assets. In particular, our tail risk measure is designed to increase with negative shocks to the aggregate market returns. Thus, we should expect that assets whose payoffs are high when our tail risk measure is high should provide insurances against severe stock market movements.

To investigate if this link materializes, that is if our tail risk measure is priced, we perform a sorting portfolio exercise using the cross-section of returns of 1228 stocks that compose the S&P 500 and Russel 1000 index from 01/02/2008 to 01/07/2015. These stocks account for a significant portion of the total market capitalization of the entire cross-section of US stocks available in the Center for Research in Securities Prices (CRSP) database[24].

We test the hypothesis of tail risk being priced by sorting portfolios on tail risk beta. We divide our analysis into two steps. First, we perform univariate sorts based on tail risk and make use of both the Fama & French and the Fama-French-Cahart linear models in trying to explain the excess returns of the sorted portfolios. Next, as in Ang, Hodrick, Xing, and Zhang (2006), Kelly and Jiang (2014), and Bollerslev et al. (2016), we use several selected stock characteristics to perform double sorts, with sorting pairs including each one of these characteristics and the tail risk beta[25].

[24]The sample considered here is comparable to that in Bollerslev et al. (2016) who select the components of the S&P 500 index from 1993 to 2010.

[25]See Appendix A for a detailed description on the control variables used in the sorting procedures.

6.1 Univariate Sorts

To compute the univariate portfolio sorts we first need to measure the hedging capacity or insurance value of all the stocks in the cross-section. Resembling the approach of Ang, Chen, and Xing (2006) and van Oordt and Zhou (2013), we estimate the tail risk beta as follows:

$$\beta_{i,TR} = \frac{cov(R_{i,t}, TR_t)}{\sqrt{var(TR_t)}}. \quad (13)$$

We rely on daily overlapping rolling-window regressions using the previous 252 trading date returns to compute the tail risk beta. Given the estimated tail risk betas, we assign stocks into ten portfolios from the portfolio with the lowest up to the one with the highest hedging ability. Given the portfolios' compositions, we compute the post-formation returns associated with them over the next day, week (five days) and month (21 days).

Table 13 reports three sets of results, each indicating a different holding period for the post-formation portfolio returns. The last three lines present the High minus Low beta portfolio returns followed by the respective risk-adjusted returns for this portfolio. Risk-adjusted returns are the intercepts of time series regression that control for either the Fama & French three-factor model or the Fama-Frech-Cahart four factor model.

First, for all three holding periods considered, we find a strong monotonic relationship between the beta sorted portfolios and excess returns. In particular, the post-formation returns range from 3.17% to 3.35% per month for a one-week and one-month holding period for the portfolio formed by stocks with the most negative tail risk betas. The returns then decay monotonically, reaching approximately 1% per month for all three holding periods for the portfolios less exposed to tail risk (the insurance portfolios). In fact, the high-minus-low portfolio has a statistically significant average return of approximately -2.21% across the different holding periods. Moreover, while lower than the unadjusted returns, the estimated alphas for both FF3 and FF4 models vary from -0.72 to -1.30 and are all statistically significant except for the FF3 alpha for the one-month holding period.

A time-series analysis of our tail risk measure shows that it is moderately persistent (with an AR(1) coefficient of 0.48). For robustness purposes, we follow Ang, Hodrick, Xing, and Zhang (2006) and re-calculate portfolio returns by sorting on tail risk innovations[26]. Tail risk surprises are measured as 1.65 standard deviations from their mean. Moreover, as an additional robustness test, we re-estimate our tail risk measure with one-minute intra-day data and re-calculate tail risk betas with this alternative series.

Table 14 presents results for the univariate sorting procedure for a one-day holding period with the alternative tail risk series. Overall, the main conclusions are the same as the ones reported for the baseline tail risk measure. Our results are also broadly consistent with the literature that links downside risks to the cross-section of expected returns. In particular, high-minus-low returns exhibit similar magnitudes to those found in previous studies such as Ang, Chen, and Xing (2006), Bollerslev et al. (2016), and Almeida et al. (2017).

So far this section reveals that stocks with lower tail risk betas tend to have higher returns, while stocks with higher tail risk betas tend to have lower returns. Nonetheless, section 4.2 provides evidence that firms with different characteristics react differently with respect to our tail risk measure. We therefore investigate in the next section whether or not the variation in returns across tail-risk beta-sorted portfolios is captured by firms characteristics that help explain the cross-section of returns.

6.1.1 Bi-variate Sorts

To investigate the pricing properties of our tail risk beta when simultaneously controlling for additional firm-level characteristics, we select several previously documented characteristics that helps explain the cross-section of stock returns: size, following Fama and French (1993); momentum, following Jegadeesh and Titman (1993); monthly, weekly and daily returns reversals, following Jegadeesh (1990); illiquidity, following Amihud (2002); turnover, following Bali et al. (2016); coskewness, following Harvey and Siddique (2000); cokurtosis, and downside beta, following Ang, Chen, and Xing (2006); downside sigma, following Da et al. (2014); idiosyncratic volatility, following Ang, Hodrick, Xing, and Zhang (2006) and max monthly return, following Bali et al. (2011).

To implement the double sorts, we first sort stocks into five quantiles according to a given firm characteristic. Then, within each quantile, we sort the firms with respect to their tail risk beta into additional five quantiles. This procedure generates 25 portfolios sorted by tail risk beta and each given firm characteristic. To construct portfolios that

[26] We measure innovations as the difference between current tail risk and an AR(7) model (the "best" time series model for tail risk) as well as an AR(1) model to be parsimonious.

are heterogeneous in the characteristic but homogeneous in tail risk beta we average the returns on the beta-sorted quantiles across the firm characteristic. By doing this, we are left with tail risk beta-sorted portfolios that have a small variation in the control variable. We focus our analysis on one-day post formation holding period.

Table 15 presents the results for the double sorts, where the last sort is in the tail risk beta. Each line indicates the control variable adopted while columns indicate the portfolios returns. The last column presents the FF4 risk-adjusted alphas for the High minus Low portfolio formed on tail risk beta. Similar to the findings for the univariate sorts, we note a monotonic relationship in the portfolio returns. While this is true for all the control variables adopted, a reasonable heterogeneity in the magnitude of the high-minus-low returns appears. The lowest value for the risk-adjusted returns is -0.84%, when we control for momentum, while the highest value is -1.74%, when the sorting variable is daily reversal. Still, all the returns for the high-minus-low portfolios are statistically significant and translate into economically meaningful returns. In fact, at a 10% significance level, with two exceptions for momentum and turnover, all the risk-adjusted alphas are negative and statistically significant.

As in Bollerslev et al. (2016) we investigate if the tail risk beta is capable of capturing previously documented anomalies by relying on reverse double sorts. Given that for most characteristics in Table 15 the excess returns on tail risk beta remains significant, we investigate the converse hypothesis: how much can the tail-risk beta explain of the previously documented anomalies? Table 16 presents these results. Similarly to Bollerslev et al. (2016), when we control for tail risk beta, several of the previously documented anomalies have a risk-adjusted return equal to zero. In fact, for momentum and turnover - the two characteristics for which the high minus low alphas were not significant in Table 15 - both the high-minus-low returns and alphas are statistically zero. Similarly to Bollerslev et al. (2016) we also find a high negative alpha related to the size effect and a high positive alpha related to the illiquidity effect. Additionally, all the reversal effects are statistically significant with monthly reversals having the same magnitude as in Bollerslev et al. (2016), and both weekly and daily reversals featuring higher alphas.

Finally, an interesting result with respect to the downside beta is noted: while the high-minus-low alpha for the tail risk beta sorted portfolio remains negative and statistically significant in the double sorts controlling for downside beta the converse is not true.

In other words, after controlling for the tail risk beta we find that the downside anomaly dissipates providing evidence that investors may price extreme market movements instead of the downside-beta characteristic.

7 A Comparative Analysis of our Tail Risk Measure

The central tenet of our tail risk measure is the non-parametric risk adjustment of the high-frequency returns of the S & P 500 index. This serves the main purpose of modifying the probabilities attached to the returns to account for the risk aversion of the investors without having to use option prices. In this section we want to show the role played by the risk adjustment in the empirical performance of the tail risk measure as well as the additional information that option prices could have brought in building the measure. We also compare methodologically our measure to the Bollerslev and Todorov (2011) investor fears index.

7.1 The Tail Risk Measure under Objective Probabilities

In this section, we build our tail risk measure with the raw returns. We define the market excess objective expected shortfall as follows:

$$TR_t = E^{\mathcal{P}_t(R)}[(VaR_\alpha(R_\tau) - R_\tau)^+] \tag{14}$$

where t is the day for which we are calculating the tail risk, τ denotes the possible states of nature (defined as the intra-day, realized index returns), α is the VaR threshold and $\mathcal{P}_t(R)$ indicates the objective density as a function of the high-frequency returns at day t. For the estimation, we proceed as described in section 2.3 except that we use the raw returns to make up the 20% tail of the distribution in order to compute the expected shortfall.

In Table 1 we report the correlations of the expected shortfall with the various measures of tail risk in the literature. These correlations are everywhere lower than the correlations computed with the risk-adjusted Hellinger tail risk measure. In Figure ..., we plot the difference between the objective tail risk measure based on the expected shortfall and the Hellinger risk-neutralized measure. The series appears to be mostly positive with several very large peaks. To rationalize this fact, recall the dual portfolio problem in equation 4 that we solve to obtain the risk-neutral probabilities. The solution to this portfolio problem for the investor is to short-sell the market in crash periods, with the effect of smoothing out the risk-neutralized tail risk measure. On the contrary the crash periods affect directly the objective expected shortfall and is fully reflected in the tail risk measure, hence the much larger peaks associated with these periods.

In terms of the respective empirical properties of the objective and risk-neutralized tail risk measures, we focus on their relations with stock market jumps and option portfolio returns. In Table 17, we provide the contemporaneous relations between the aggregate jump measure and the expected shortfall, which are to be compared with the similar results for the Hellinger tail risk measure in Table 2. The R^2 and $Student - t$ statistics are always higher for the tail risk measure than for the expected shortfall but the more interesting result is for the VIX. In the tail risk regressions, as we stressed in section 3, the VIX has a strong and significant negative effect on the number of jumps. In other words we are to separate the tail effects from the variance effects. It is not the case with the expected shortfall since it amalgamates the continuous and jump variations. The sign for VIX is negative but the effect is nowhere significant.

The expected shortfall results are similar for the weekly downside jump prediction regressions in Table 18, to be compared with the tail risk results in Table 6. The R^{2*} statistic shows undeniably that tail risk predicts better future weekly jumps than the expected shortfall. Moreover the effect is statistically distinguishable from the VIX for all sizes of jumps while it is the case only for the very large jumps for the expected shortfall.

Finally, for the options portfolios prediction regressions reported in Table 19 for the expected shortfall and to be compared to Table 12 for the Hellinger tail risk measure, we observe that the tail risk measure is a good predictor for all portfolios of options while the expected shortfall does not predict well the out-of-the-money (OTM) and deep OTM portfolios. Moreover it contributes less to the R^2 in the multivariate regressions than th tail risk. Therefore one can conclude that the Hellinger risk-adjustment is better able to capture the downside risk than the expected shortfall[27].

[27] We do not report the results for the expected shortfall for the relations with the stock index and Treasury returns and the pricing of portfolios since they are very similar to the ones obtained with the Hellinger tail risk measure. When significant the effects are a bit stronger for the expected shortfall.

7.2 The Tail Risk Measure with High-Frequency Option Returns

Stutzer (1996) has shown that tilting the physical probability measure with an entropic loss provides a useful tool to price options. In this paper we adopt a measure similar to the entropy, the Hellinger measure, which is part of a more general family of entropic measures, the Cressie Read family. The main innovation here is the use of high-frequency returns instead of data at lower frequencies (daily, monthly). The tilting of our Cressie Read Hellinger loss function offers a way to capture, in principle, the risk neutralization that options provide when estimating a tail risk measure. In fact, Almeida and Freire (2020) show that the Cressie Read family provides economically motivated option pricing bounds based on pseudo investors holding only a stock index and a riskless asset, which are robust to different levels of risk-premia of stochastic volatility and jumps.

Despite the strong theoretical properties that the Cressie Read family presents when estimating risk neutral measures based only on the S&P 500 returns, it will be informative to include S&P 500 option returns in the estimation of the Hellinger risk neutral measure that generates our tail risk measure. We intend to use high-frequency option data to estimate an alternative tail risk measure and to compare it to our benchmark measure that uses only S&P 500 high-frequency returns.

[**Data description, new estimation equation and empirical results to be added**]

7.3 A Comparison with the Bollerslev and Todorov (2011) Nonparametric Measure

Bollerslev and Todorov (2011) start with an Ito semimartingale structure for the future price of the aggregate market portfolio under the physical measure: $\frac{dF_t}{F_t} = \alpha_t dt + \sigma_t dW_t + \int_{\Re}(e^x - 1)\tilde{\mu}(dt, dx)$, with $\tilde{\mu}(dt, dx)$ being a counting measure with compensator $\nu_t^P(dx)dt$. From this structured dynamics for the index future price, which has the same structure under the risk neutral measure, they use small time asymptotics to relate the prices of short-maturity out-of-the-money puts (calls) to a measure of left (right) tail risk that is based on the risk neutral compensator $\nu_t^Q(dx)dt$ of the index future price dynamics. In fact, the options are used to identify this compensator. Additionally, giving further structure to the compensator ($\nu_t^Q(dx) = (\phi_t^+ 1_{\{x>0\}} + \phi_t^- 1_{\{x<0\}})\nu^Q(x)dx$) and relying on extreme value theory to approximate the tail of the risk neutral distribution by a Pareto distribution, they are able to relate their tail risk measures to the equity and variance risk premia.

In contrast, in this paper, we concentrate on the risk neutral expected shortfall function $max(k_t - x, 0)$ to obtain a nonparametric tail risk measure without suggesting any structure for the dynamics of the returns of the index. While Bollerslev and Todorov (2011) risk neutral measure has separate components for the diffusive and jump parts we keep those entangled and obtain from our Hellinger loss function a hyperbolic risk neutral measure that has a thick tail. Instead of adopting options and a parametric risk neutral compensator for the jump measure, we tilt the physical measure based on our Hellinger loss function to obtain the nonparametric risk-neutral measure that is used to estimate the tail risk measure as the price of a synthetic out-of-the-money put option.

8 Conclusion

We propose a new measure of tail risk based on high-frequency risk-neutralized market returns. This measure presents useful characteristics that contribute to the tail risk literature. First, it can be estimated on a daily basis without using overlapping data based only on intraday returns. Second, its nonparametric approach to risk neutralization offers quick reactions to changes in market conditions. Third, and perhaps most importantly, the measure does not depend on option prices and allows for possible extensions of our analysis to individual assets and to any market where high-frequency recordings of prices are available.

Our tail risk series computed from 2008 to 2016 captures the 2008 financial crisis, the 2011 European debt crisis, and has a significant correlation with financial, uncertainty, and macroeconomic measures revealing a clear relationship between stock market fluctuations and the real economy.

Our extensive analysis of the relationship between different assets' returns and tail risk reveals not only that several asset classes are exposed to market tail risk but also that some strategies provide a significant hedge against risks of extreme market return realizations. In particular, we document a strong fly-to-safety and mean reversion feature of Treasury

ETFs, a strong negative relationship between various risky assets returns and tail risk, and the tail risk implications to expected returns on the cross-section of assets. We also show that tail risk is closely related to put options returns, with long-short strategies on deep out-of-the-money puts and deep in-the-money puts providing a significant hedge against tail risk.

Appendix A:

In this appendix, we give a brief description of the additional control variables we consider in the double sort procedures.

- SIZE: Following Fama and French (1993) firm size is measured at the end of each June by its market value, defined as the stock price multiplied by the number of shares outstanding. We update SIZE annually and use it to explain the following 12 months returns. If a stock is introduced in our dataset after the June cut-off we define its size as the stock price multiplied by the shares outstanding for the first day the stock appears in the data set and repeat this value until the next June breaking point. Following Bollerslev et al. (2016) the final value for SIZE is the natural logarithm of the firms' size.

- MOMENTUM: Following Jegadeesh and Titman (1993) we measure momentum as the gross return of the last 252 trading dates skipping the short term reversal, here defined as the last 21 trading dates. If, for a given date t, we do not have the last 252 days returns for a given stock than we calculate momentum using the maximum observable returns in our sample. If this is less than 50 trading days, we discard this stick for date t cross-section.

- REVERSAL: We calculate three reversal variables following Jegadeesh (1990) and Bali et al. (2016).
  - Monthly Reversal: Is defined as the aggregate return of the last 21 trading days.
  - Weekly Reversal: Is defined as the aggregate return of the last 5 trading days.
  - Daily Reversal: Is defined as the last trading day return.

- ILLIQUIDITY: Following Amihud (2002) we define illiquidity for each stock i at date t as follows:

$$ILLIQ_{i,t} = \frac{1}{T}\sum_{i=1}^{T}\frac{|r_{i,t}|}{volume_{i,t}price_{i,t}} \tag{15}$$

  For each data t we use data for the preceding five trading days to calculate illiquidty. $volume_{i,t}$ is stock i date t trading volume, $price_{i,t}$ is stock i date t closing price.

- TURNOVER: Alternatively to illiquidity, following Bali et al. (2016), we define turnover as the average ratio between daily trades and total shares outstanding for the past five days.

- COSKEWNESS: Following Harvey and Siddique (2000), the daily firm co-skewness is defined as the β_{CS} estimate using daily data for the preceding 21 trading dates for asset i for the following regression:

$$R_{i,t} - Rf_t = \alpha + \beta_{MKT}(R_{m,t} - Rf_t) + \beta_{CS}((R_{m,t} - Rf_t))^2 + \epsilon_{i,t} \tag{16}$$

  Where $R_{i,t}$, Rf_t and $R_{m,t}$ denote the asset i date t returns, the risk free rate and the market return respectively.

- COKURTOSIS: Similar to Ang, Chen, and Xing (2006), the daily firm co-kurtosis is defined as the β_{CK} estimate using daily data for the preceding 21 trading dates for asset i for the following regression:

$$R_{i,t} - Rf_t = \alpha + \beta_{MKT}(R_{m,t} - Rf_t) + \beta_{CS}((R_{m,t} - Rf_t))^2 + \beta_{CK}((R_{m,t} - Rf_t))^3 + \epsilon_{i,t} \tag{17}$$

  Where $R_{i,t}$, Rf_t and $R_{m,t}$ denote the asset i date t returns, the risk free rate and the market return respectively.

- DOWNSIDE BETA: Following Ang, Chen, and Xing (2006), the daily downside beta is defined as follows:

$$\beta_i^- = \frac{cov(R_i, R_m | R_m < \mu_m)}{var(R_m | R_m < \mu_m)} \tag{18}$$

Where R_i, R_m, μ_m denote the asset i returns, the market returns and the market mean. We use the last 252 trading dates returns to calculate the downside beta. As for MOMENTUM, if, for a given date t, we do not have the last 252 days returns for a given stock than we calculate momentum using the maximum observable returns in our sample. If this is less than 50 trading days we discard this stick for date t cross-section.

- DOWNSIDE SIGMA: Following Da et al. (2014), the daily downside sigma is defined as follows:

$$\sigma_i^- = \sqrt{var(R_i | R_m < \mu_m)} \tag{19}$$

Where R_i, R_m, μ_m denote the asset i returns, the market returns and the market mean. We use the last 252 trading dates returns to calculate the downside beta. As for MOMENTUM, if, for a given date t, we do not have the last 252 days returns for a given stock than we calculate momentum using the maximum observable returns in our sample. If this is less than 50 trading days we discard this stick for date t cross-section.

- IVOL: Following Ang, Hodrick, Xing, and Zhang (2006), the daily firm idiosyncratic volatility is compute as the standard deviation for the residuals from the following regression for the last 21 trading days:

$$R_{i,t} - Rf_t = \alpha + \beta_{MKT}(R_{m,t} - Rf_t) + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \epsilon_{i,t} \tag{20}$$

Where $R_{i,t}$, Rf_t, $R_{m,t}$, SMB_t and HML_t denote the asset i date t returns, the risk free rate, the market return, the SMB_t and HML_t portfolios of Fama and French (1993) respectively.

- MAX: Following Bali et al. (2011) MAX is defined as the maximum return of a given firm in the preceding 21 trading days.

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Table 1: **Correlations**

	Weller daily	Weller 10-day MA	VIX	RV	BTX	EPU	Eq. EPU
Hellinger Tail Risk	0.64	0.56	0.65	0.65	0.59	0.34	0.41
Expected Shortfall	0.55	0.50	0.61	0.76	0.51	0.29	0.40
Moving Average 10 Days	0.76	0.84	0.90	0.68	0.68	0.43	0.47

	ADS	Noise	Spread	Crashes	Fin. Beta	Yield Spread	FEAR
Hellinger Tail Risk	-0.42	0.49	0.47	0.32	0.35	0.48	0.07
Expected Shortfall	-0.37	0.44	0.42	0.29	0.31	0.43	0.12
Moving Average 10 Days	-0.60	0.73	0.71	0.50	0.52	0.72	-0.05

This table presents the correlation coefficients between the Hellinger Tail Risk, its physical counterpart, the 10-day moving average of the Hellinger Tail Risk and various other crash sensitive measures. The Weller daily and 10-day moving average (MA) are as described in section 2.3. VIX indicates the CBOE VIX index; RV stands for realized variance - measured as the squared sum of intraday 15-min S&P 500 returns; BTX stands for the the left risk neutral jump tail variation of Bollerslev et al. (2015); EPU and Eq. EPU are the Economic Policy Uncertainty Index and Equity Uncertainty Index for the U.S. economy from Baker et al. (2016). ADS stands for the the Aruoba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB; Noise stands for the noise measure of Hu et al. (2013); Spread stands for the Emerging Markets spread with the spot treasury curve from BofA (Available at St. Louis FRED); Crashes stands for the Stock Market Crashes measured by the Cleveland FRB as the ratio of the current value for the S&P 500 index and its maximum over the last 365 days; Fin. Beta stands for the measure of the contribution of the banking sector to overall stock market volatility from the Cleveland FRB; Yield Spread stands for the difference between the Moody's Seasoned Baa Corporate Bond and the 10-Year Treasury Constant Maturity from St. Louis FRED; FEAR stands for the Google Trends measure of investors sentiment from Da et al. (2014).

Table 2: **Contemporaneous Downside Jump Regressions**

	Tail Risk	VIX	RV	EPU	ADS
	Panel A: Univariate Regressions				
β	0.11	0.04	0.07	0.03	-0.01
$t-stat.$	(6.69)	(3.55)	(3.41)	(2.62)	(-0.78)
R^2	0.16	0.03	0.07	0.01	0.00
	Panel B: Multivariate Regressions				
Tail Risk		0.13	0.12	0.12	0.12
		(7.70)	(7.50)	(7.45)	(7.58)
VIX		-0.04	-0.05	-0.05	-0.03
		(-4.18)	(-4.38)	(-4.46)	(-2.12)
RV			0.03	0.03	0.02
			(1.49)	(1.49)	(1.42)
EPU				0.00	0.00
				(0.37)	(0.29)
ADS					0.00
					(0.29)
R^2		0.17	0.17	0.17	0.18

This table presents results for contemporaneous regressions where the endogenous variable is the Downside Jump measure calculated in section 3. Panel A features the results of univariate regressions where the explanatory variable is in turn the Hellinger Tail Risk, the CBOE VIX index, the realized variance - measured as the squared sum of intraday 15-min S&P 500 returns, the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016), and the Aruoba, Diebold and Scotti (ADS) Business Conditions Index. Panel B presents multivariate regressions in addition to our tail risk measure, we include as controls the above-mentioned crash sensitive indexes one at a time. The t-statistics are calculated using a Newey-West variance matrix with five lags.

Table 3: **Contemporaneous Downside Jump Regressions - Segregated**

	D. Jump 1	D. Jump 2	D. Jump 3	D. Jump 4	D. Jump 5	Aggregate
Tail Risk	0.26	0.24	0.11	0.07	0.10	0.12
	(4.39)	(4.99)	(5.04)	(4.30)	(5.75)	(7.58)
VIX	0.07	-0.08	-0.04	-0.03	-0.03	-0.03
	(0.72)	(-1.96)	(-2.20)	(-1.92)	(-2.46)	(-2.12)
RV	0.13	0.05	0.02	0.00	0.01	0.02
	(2.01)	(1.11)	(1.10)	(0.42)	(0.63)	(1.42)
EPU	0.04	-0.02	0.01	0.01	0.00	0.00
	(0.91)	(-0.78)	(0.79)	(0.70)	(-0.43)	(0.29)
ADS	0.11	0.04	0.04	0.01	0.03	0.03
	(1.23)	(1.12)	(2.33)	(0.51)	(2.17)	(1.92)
Constant	1.99	0.43	0.12	0.05	0.07	0.25
	(35.67)	(18.30)	(11.79)	(9.28)	(10.50)	(28.67)
R^2	0.05	0.08	0.08	0.06	0.09	0.18
R^{2*}	0.04	0.06	0.06	0.05	0.07	0.16

This table presents results for contemporaneous regressions where the endogenous variable is the Downside Jump measure calculated in section 3 and its five components. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance - measured as the squared sum of intraday 15 min S&P 500 returns, the Aruoba, Diebold and Scotti Business Conditions Index, and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). R^2* denotes the coefficient of determination obtained from a univariate regression where the only explanatory variable is the Hellinger Tail Risk. The t-statistics are calculated using Newey-West variance matrix with five lags.

Table 4: **Downside Jump Predictive Regressions**

	Tail Risk	VIX	RV	EPU	ADS
	Panel A: Bivariate Regressions				
β	0.02	0.01	0.03	0.01	-0.01
$t - stat.$	(3.03)	(1.43)	(4.25)	(0.88)	(-0.75)
R^2	0.22	0.22	0.23	0.22	0.22
	Panel B: Multivariate Regressions				
Tail Risk		0.03	0.02	0.02	0.02
		(3.11)	(2.15)	(2.14)	(2.14)
VIX		-0.01	-0.02	-0.02	-0.02
		(-1.50)	(-2.69)	(-2.77)	(-2.29)
RV			0.03	0.03	0.03
			(4.37)	(4.37)	(4.42)
EPU				0.00	0.00
				(0.19)	(0.19)
ADS					0.00
					(-0.08)
R^2		0.22	0.23	0.23	0.23

This table presents results for predictive regressions where the endogenous variable is the Downside Jump measure calculated in section 3. Panel A features the results for bi-variate regressions where, in addition to the lag of the realized jump measure, we consider in turn as explanatory variable the Hellinger Tail Risk, the CBOE VIX index, the realized variance - measured as the squared sum of intraday 15 min S&P 500 returns, the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016), and the Aruoba, Diebold and Scotti Business Conditions Index. Panel B presents multivariate regressions with our tail risk measure and controls including the above-mentioned crash sensitive indexes. The t-statistics are calculated using a Newey-West variance matrix with five lags.

Table 5: **Downside Jump Predictive Regressions - Segregated**

	D. Jump 1	D. Jump 2	D. Jump 3	D. Jump 4	D. Jump 5	Aggregate
Tail Risk	0.26	0.07	0.05	0.02	0.04	0.02
	(4.49)	(3.12)	(2.73)	(1.69)	(2.86)	(2.14)
VIX	-0.11	-0.03	-0.02	-0.02	-0.03	-0.02
	(-1.56)	(-0.92)	(-1.09)	(-1.99)	(-2.87)	(-2.29)
RV	0.05	0.04	0.03	0.03	0.03	0.03
	(1.04)	(2.17)	(1.22)	(1.49)	(2.65)	(4.42)
EPU	0.05	-0.03	0.00	0.01	0.00	0.00
	(1.01)	(-1.53)	(-0.20)	(1.00)	(0.13)	(0.19)
ADS	-0.01	0.01	0.02	0.00	0.01	0.00
	(-0.14)	(0.21)	(1.55)	(-0.19)	(0.80)	(-0.08)
Lag Jump	0.27	0.25	0.14	0.00	0.07	0.42
	(11.07)	(7.00)	(2.26)	(0.09)	(1.61)	(9.94)
Constant	1.45	0.32	0.10	0.05	0.06	0.14
	(23.51)	(15.46)	(11.31)	(8.45)	(9.99)	(14.49)
R^2	0.11	0.09	0.05	0.02	0.04	0.23
R^2*	0.04	0.02	0.02	0.01	0.02	0.07

This table presents results for predictive regressions where the endogenous variable is the Downside Jump measure calculated in section 3 and its five components. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance - measured as the squared sum of intraday 15 min S&P 500 returns, the Aruoba, Diebold and Scotti (ADS) Business Conditions Index, and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). R^2* denotes the coefficient of determination obtained from a univariate regression where the only explanatory variable is the Hellinger Tail Risk. The t-statistics are calculated using a Newey-West variance matrix with five lags.

Table 6: **Weekly Downside Jump Predictive Regressions - Segregated**

	D. Jump 1	D. Jump 2	D. Jump 3	D. Jump 4	D. Jump 5	Aggregate
Tail Risk	1.13	0.51	0.24	0.23	0.12	0.20
	(2.92)	(2.19)	(2.00)	(3.52)	(2.08)	(2.90)
VIX	-0.68	-0.49	-0.14	-0.11	-0.15	-0.24
	(-1.57)	(-2.26)	(-1.58)	(-1.75)	(-2.55)	(-3.05)
RV	-0.30	-0.06	0.01	0.02	0.03	-0.03
	(-2.29)	(-0.65)	(0.14)	(0.38)	(1.18)	(-1.02)
EPU	-0.11	-0.01	0.03	-0.02	0.03	0.02
	(-0.29)	(-0.07)	(0.35)	(-0.59)	(0.48)	(0.29)
ADS	-0.31	-0.16	0.05	0.00	-0.02	-0.09
	(-0.70)	(-0.93)	(0.73)	(-0.09)	(-0.39)	(-1.35)
Lag Jump	0.36	0.32	0.19	0.14	0.25	0.51
	(6.21)	(5.90)	(3.27)	(1.91)	(3.65)	(9.61)
Constant	6.07	1.39	0.45	0.21	0.25	0.59
	(9.52)	(9.19)	(7.56)	(6.97)	(7.13)	(8.89)
R^2	0.17	0.14	0.08	0.13	0.10	0.31
R^2*	0.04	0.03	0.03	0.09	0.02	0.07

This table presents results for weekly predictive regressions where the endogenous variable is the Downside Jump measure calculated in section 3 and its five components. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance - measured as the squared sum of intraday 15 min S&P 500 returns, the Aruoba, Diebold and Scotti (ADS) Business Conditions Index and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). In the last line we also present the R^2* for univariate regressios where the only explanatory variable is the Hellinger Tail Risk. The t-statistics are calculated using a Newey-West variance matrix with five lags.

Table 7: **S&P 500 Regressions**

	$S\&P500_t$	$S\&P500_{t+1}$	$S\&P500_{t+2}$
TR_t	-0.51	-0.30	0.15
	(-4.62)	(-2.42)	(1.69)
VIX_t	-0.17	0.28	-0.05
	(-1.60)	(3.09)	(-0.60)
$Variance_t$	0.29	0.11	0.08
	(2.18)	(0.86)	(1.12)
EPU_t	0.13	-0.04	0.01
	(2.79)	(-0.83)	(0.13)
ADS_t	-0.12	0.16	0.12
	(-2.43)	(3.05)	(2.32)
Cons	0.03	0.03	0.03
	(1.02)	(1.06)	(1.03)
$S\&P500_t$		-0.13	-0.05
		(-3.46)	(-0.87)
R^2	0.09	0.04	0.02
R^2*	0.06	0.00	0.01

This table present the results for contemporaneous and predictive regressions where the endogenous variable is the daily return on the S&P 500 index. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance - measured as the squared sum of intraday 15 min S&P 500 returns, the Aruoba, Diebold and Scotti Business Conditions Index and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). In the last line we also present the R^2* for univariate regressions where the only explanatory variable is the Hellinger Tail Risk. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 8: **Treasury Regressions**

	$Treasury_t$	$Treasury_{t+1}$	$Treasury_{t+2}$
TR_t	0.12	0.01	-0.04
	(5.45)	(0.42)	(-2.22)
VIX_t	0.05	0.00	0.06
	(1.73)	(-0.09)	(2.70)
$Variance_t$	-0.07	-0.01	-0.02
	(-2.12)	(-0.47)	(-1.38)
EPU_t	-0.04	0.01	0.00
	(-3.10)	(0.66)	(-0.12)
ADS_t	0.04	0.00	0.01
	(2.64)	(0.01)	(0.87)
Cons	0.01	0.01	0.01
	(1.09)	(1.05)	(1.04)
$Treasury_t$		-0.05	-0.03
		(-2.11)	(-1.27)
R^2	0.04	0.00	0.01
R^2*	0.02	0.00	0.00

This table present the results for contemporaneous and predictive regressions where the endogenous variable is the daily return on the iShares Treasury Bond ETF. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance - measured as the squared sum of intraday 15 min S&P 500 returns, the Aruoba, Diebold and Scotti Business Conditions Index and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). In the last line we also present the R^2* for univariate regressions where the only explanatory variable is the Hellinger Tail Risk. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 9: **Beta Sorted Portfolio Regressions**

Portfolio	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	High - Low
	Panel A: Contemporaneous Regressions										
SPX	-0.75	-0.65	-0.59	-0.52	-0.49	-0.44	-0.40	-0.34	-0.17	-0.04	0.71
	(-4.33)	(-4.41)	(-4.57)	(-4.53)	(-4.54)	(-4.61)	(-4.45)	(-4.64)	(-3.20)	(-1.06)	(4.31)
VIX	-0.31	-0.27	-0.24	-0.21	-0.18	-0.15	-0.13	-0.08	-0.04	0.02	0.33
	(-1.61)	(-1.65)	(-1.68)	(-1.62)	(-1.51)	(-1.41)	(-1.33)	(-1.02)	(-0.56)	(0.42)	(1.86)
RV	0.41	0.33	0.31	0.25	0.24	0.19	0.16	0.08	-0.03	-0.10	-0.51
	(2.30)	(2.18)	(2.24)	(2.16)	(2.19)	(1.94)	(1.83)	(1.28)	(-0.57)	(-1.58)	(-2.33)
EPU	0.24	0.21	0.18	0.15	0.13	0.13	0.10	0.08	0.05	0.04	-0.21
	(2.95)	(3.10)	(3.05)	(2.86)	(2.52)	(2.85)	(2.53)	(2.39)	(2.06)	(2.24)	(-2.64)
ADS	-0.29	-0.23	-0.21	-0.18	-0.16	-0.14	-0.13	-0.11	-0.08	-0.05	0.23
	(-2.59)	(-2.68)	(-2.76)	(-2.70)	(-2.63)	(-2.64)	(-2.57)	(-2.50)	(-2.19)	(-1.76)	(2.42)
R^2	0.06	0.07	0.08	0.07	0.07	0.08	0.08	0.08	0.06	0.04	0.07
	Panel B: Predictive Regressions										
SPX	-0.44	-0.38	-0.36	-0.33	-0.33	-0.29	-0.27	-0.23	-0.18	-0.11	0.34
	(-2.45)	(-2.38)	(-2.46)	(-2.34)	(-2.39)	(-2.36)	(-2.23)	(-1.96)	(-1.86)	(-2.22)	(2.27)
VIX	0.52	0.40	0.36	0.32	0.30	0.26	0.23	0.17	0.12	0.06	-0.45
	(3.39)	(3.11)	(3.12)	(2.92)	(3.00)	(2.93)	(2.77)	(2.24)	(2.14)	(1.53)	(-3.24)
RV	0.09	0.09	0.09	0.10	0.11	0.10	0.10	0.11	0.10	0.07	-0.05
	(0.51)	(0.56)	(0.60)	(0.72)	(0.79)	(0.76)	(0.82)	(0.82)	(0.84)	(1.19)	(-0.33)
EPU	-0.03	-0.02	-0.02	-0.02	-0.03	-0.02	-0.02	-0.02	-0.03	-0.01	0.03
	(-0.36)	(-0.33)	(-0.32)	(-0.37)	(-0.50)	(-0.44)	(-0.44)	(-0.53)	(-1.13)	(-0.57)	(0.33)
ADS	0.20	0.17	0.15	0.14	0.13	0.12	0.11	0.09	0.05	0.01	-0.20
	(1.85)	(2.02)	(1.97)	(2.12)	(2.17)	(2.23)	(2.27)	(2.12)	(1.66)	(0.23)	(-2.05)
Lag Ret.	0.02	-0.01	-0.04	-0.02	-0.04	-0.04	-0.02	0.00	0.09	0.23	0.01
	(0.51)	(-0.28)	(-1.03)	(-0.49)	(-1.03)	(-0.92)	(-0.51)	(0.06)	(1.59)	(6.52)	(0.14)
R^2	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.04	0.07	0.02

This table present the results for contemporaneous (Panel A) and predictive (Panel B) regressions for the NYSE/AMEX Market Beta sorted portfolios from CRSP. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance - measured as the squared sum of intraday 15-minutes S&P 500 returns, the Aruoba, Diebold and Scotti Business Conditions Index, the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016) and lagged returns for each specific dependent variable. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 10: **Variance-Sorted Portfolio Regressions**

Portfolio	1	2	3	4	5	6	7	8	9	10	High - Low
	Panel A: Contemporaneous Regressions										
SPX	-0.51	-0.58	-0.56	-0.57	-0.53	-0.49	-0.44	-0.35	-0.26	-0.08	0.43
	(-4.36)	(-4.69)	(-4.53)	(-4.53)	(-4.41)	(-4.23)	(-4.42)	(-4.32)	(-4.00)	(-1.85)	(4.60)
VIX	-0.17	-0.21	-0.24	-0.23	-0.22	-0.21	-0.16	-0.10	-0.05	0.02	0.19
	(-1.29)	(-1.47)	(-1.68)	(-1.63)	(-1.67)	(-1.65)	(-1.48)	(-1.18)	(-0.73)	(0.45)	(1.75)
RV	0.13	0.25	0.30	0.31	0.28	0.25	0.20	0.11	0.04	-0.08	-0.21
	(1.70)	(2.11)	(2.24)	(2.24)	(2.17)	(2.12)	(2.02)	(1.46)	(0.61)	(-1.13)	(-2.51)
EPU	0.17	0.18	0.18	0.17	0.16	0.15	0.12	0.09	0.07	0.03	-0.14
	(2.84)	(3.06)	(3.09)	(3.12)	(3.01)	(2.75)	(2.74)	(2.37)	(2.12)	(1.28)	(-2.54)
ADS	-0.27	-0.23	-0.20	-0.19	-0.17	-0.17	-0.14	-0.11	-0.08	-0.02	0.25
	(-2.73)	(-2.60)	(-2.58)	(-2.67)	(-2.67)	(-2.83)	(-2.76)	(-2.63)	(-2.21)	(-0.60)	(2.89)
R^2	0.06	0.07	0.07	0.07	0.08	0.08	0.08	0.08	0.07	0.04	0.05
	Panel B: Predictive Regressions										
SPX	-0.28	-0.34	-0.35	-0.36	-0.32	-0.31	-0.28	-0.24	-0.19	-0.14	0.12
	(-2.22)	(-2.45)	(-2.60)	(-2.46)	(-2.45)	(-2.37)	(-2.21)	(-2.05)	(-1.74)	(-1.54)	(1.68)
VIX	0.32	0.38	0.38	0.35	0.31	0.28	0.24	0.20	0.14	0.08	-0.24
	(2.80)	(3.21)	(3.38)	(3.14)	(3.02)	(2.76)	(2.81)	(2.53)	(2.12)	(1.58)	(-2.64)
RV	0.06	0.06	0.06	0.09	0.08	0.11	0.11	0.12	0.13	0.13	0.06
	(0.46)	(0.44)	(0.48)	(0.64)	(0.64)	(0.78)	(0.81)	(0.87)	(0.94)	(1.08)	(1.05)
EPU	-0.03	-0.01	-0.03	-0.01	-0.02	-0.02	-0.02	-0.03	-0.03	-0.02	0.01
	(-0.44)	(-0.20)	(-0.43)	(-0.25)	(-0.28)	(-0.41)	(-0.50)	(-0.86)	(-0.86)	(-1.23)	(0.17)
ADS	0.07	0.13	0.15	0.16	0.15	0.13	0.12	0.11	0.09	0.05	-0.03
	(0.85)	(1.58)	(1.97)	(2.20)	(2.34)	(2.26)	(2.46)	(2.49)	(2.42)	(2.00)	(-0.39)
Lag Ret.	0.11	0.02	-0.04	-0.06	-0.05	-0.05	-0.04	-0.00	0.08	0.22	0.14
	(3.15)	(0.65)	(-0.99)	(-1.34)	(-1.08)	(-1.15)	(-0.95)	(-0.02)	(1.38)	(3.34)	(4.10)
R^2	0.03	0.02	0.02	0.03	0.02	0.03	0.03	0.03	0.04	0.09	0.03

This table presents results for contemporaneous (Panel A) and predictive (Panel B) regressions for the NYSE/AMEX Market Beta sorted portfolios from CRSP. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance - measured as the squared sum of intraday 15-minutes S&P 500 returns, the Aruoba, Diebold and Scotti Business Conditions Index, the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016) and lagged returns for each specific dependent variable. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 11: **Additional Portfolios Regressions**

	Small	Big	SMB	High	Lo	HML	Win S.	Looser S.	WML S.	Win B.	Looser B.	WML B.
Tail Risk	-0.31	-0.29	-0.03	-0.33	-0.27	-0.06	-0.30	-0.37	0.09	-0.22	-0.39	0.17
	(-2.65)	(-2.38)	(-0.81)	(-2.45)	(-2.32)	(-1.67)	(-2.50)	(-2.55)	(2.31)	(-2.00)	(-2.68)	(2.97)
VIX	0.30	0.27	0.02	0.28	0.26	0.02	0.30	0.37	-0.06	0.27	0.38	-0.11
	(2.95)	(3.11)	(0.62)	(2.39)	(3.28)	(0.43)	(3.01)	(3.11)	(-1.45)	(3.21)	(2.95)	(-1.50)
Variance	0.02	0.11	-0.08	0.09	0.10	-0.01	0.05	0.06	-0.04	0.04	0.17	-0.12
	(0.21)	(0.87)	(-2.06)	(0.61)	(0.84)	(-0.21)	(0.39)	(0.52)	(-1.26)	(0.39)	(1.10)	(-1.69)
EPU	-0.01	-0.04	0.03	-0.01	-0.04	0.03	-0.01	-0.03	0.02	-0.03	-0.03	0.00
	(-0.13)	(0.04)	(1.26)	(-0.20)	(-0.89)	(1.05)	(-0.17)	(-0.50)	(0.95)	(-0.72)	(0.06)	(0.07)
ADS	0.14	0.16	-0.02	0.16	0.15	0.01	0.17	0.12	0.03	0.18	0.19	-0.02
	(2.07)	(3.07)	(-0.82)	(2.09)	(3.04)	(0.28)	(2.62)	(1.45)	(0.86)	(3.44)	(2.13)	(-0.26)
Cons	0.05	0.04	0.01	0.04	0.05	-0.01	0.05	0.06	-0.01	0.04	0.03	0.00
	(1.25)	(1.38)	(0.44)	(0.99)	(1.73)	(-0.40)	(1.41)	(1.27)	(-0.44)	(1.43)	(0.76)	(0.15)
Lag Return	-0.11	-0.12	-0.08	-0.09	-0.12	-0.03	-0.09	-0.02	0.18	-0.08	-0.03	0.06
	(-2.96)	(-3.21)	(-2.79)	(-2.08)	(-3.07)	(-0.65)	(-2.47)	(-0.55)	(4.96)	(-2.35)	(-0.83)	(1.43)
R^2	0.03	0.04	0.02	0.02	0.04	0.00	0.03	0.02	0.04	0.03	0.02	0.01

This table presents results for one-day ahead predictive regressions where the endogenous variables, indicated in each column, are the returns on (1) small firms portfolios, (2) big firms portfolios, (3) SMB firms portfolios, (4) high book to market firms portfolios, (5) low book to market firms portfolios, (6) HML firms portfolios, (7) double sorted winners and small firms portfolios, (8) double sorted Loosers and small firms portfolios, (9) WML small firms portfolios, (10) double sorted winners and big firms portfolios, (11) double sorted winners and big firms portfolios, (12) WML big firms portfolios. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance - measured as the squared sum of intraday 15-minutes S&P 500 returns, the Aruoba, Diebold and Scotti Business Conditions Index, the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). t-statistics are calculated using Newey-West variance matrix with five lags.

Table 12: **Options Portfolios Predictive Regressions**

	DOTM	OTM	ATM	ITM	DITM	DOTM-DITM	Corr.
			Panel A: Contemporaneous				
Tail Risk	0.09	0.08	0.07	0.05	0.04	0.05	
	(2.69)	(2.73)	(2.68)	(2.70)	(2.68)	(2.41)	
R^2	0.21	0.21	0.22	0.22	0.22	0.17	
R^2*	0.08	0.08	0.08	0.11	0.12	0.06	
			Panel B: Predictive				
Tail Risk	0.10	0.08	0.06	0.04	0.03	0.06	
	(2.60)	(2.56)	(2.05)	(2.08)	(2.11)	(2.42)	
R^2	0.04	0.03	0.03	0.04	0.06	0.02	
R^2*	0.02	0.02	0.01	0.02	0.03	0.01	
			Panel C: Factor Model				
FF3 Alpha	-0.05	-0.03	0.03	0.04	0.02	-0.07	
	(-5.31)	(-3.99)	(3.87)	(11.33)	(10.72)	(-8.74)	-0.72
FF3+MOM Alpha	-0.05	-0.04	0.03	0.04	0.02	-0.07	
	(-5.40)	(-4.05)	(3.91)	(11.53)	(10.85)	(-8.85)	-0.72

This table presents results for contemporaneous and one-day ahead predictive regressions where the endogenous variables are portfolios formed on index options according to the options moneyness for put options with expiration dates between 1 and 45 days counting from the portfolio formation date. The first three lines present the estimated beta, t-statistics and R^2 from multivariate contemporaneous regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance - measured as the squared sum of intraday 15-minutes S&P 500 returns, the Aruoba, Diebold and Scotti Business Conditions Index, the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016), and a Downside Jump measure. The fourth line presents the estimated R^2 for a regression where the Hellinger Tail Risk is the only explanatory variable. Lines 5-8 present the same statistics as lines 1-4 for predictive regressions where the lag of the portfolio return is an additional explanatory variable. Lines 9-12 present the alphas and t-statistics respectively from the Fama and French three-factor and the Fama-French-Cahart four-factor models. The last column presents the correlation between the vector of alphas and the vector of Hellinger Tail Risk betas obtained in the contemporaneous multivariate regressions. *t*-statistics are calculated using Newey-West variance matrix with five lags.

Table 13: **Tail Risk Sorted Portfolios**

Portfolio	Ret.	t	Ret.	t	Ret.	t
1.00	3.28	(2.90)	3.17	(3.22)	3.35	(2.98)
2.00	2.31	(2.54)	2.20	(2.97)	2.32	(3.16)
3.00	2.01	(2.49)	1.97	(3.02)	2.01	(3.21)
4.00	1.83	(2.46)	1.78	(2.98)	1.85	(3.28)
5.00	1.68	(2.43)	1.70	(3.11)	1.73	(3.52)
6.00	1.56	(2.44)	1.42	(2.74)	1.48	(3.24)
7.00	1.41	(2.35)	1.42	(2.94)	1.45	(3.40)
8.00	1.38	(2.51)	1.27	(2.90)	1.30	(3.32)
9.00	1.08	(2.10)	1.13	(2.71)	1.24	(3.54)
10.00	0.97	(1.83)	1.03	(2.34)	1.17	(3.09)
High - Low	-2.31	(-3.03)	-2.14	(-3.08)	-2.18	(-2.50)
FF3 Alpha	-1.30	(-2.77)	-1.11	(-2.48)	-0.72	(-1.42)
FF4 Alpha	-1.24	(-2.73)	-1.06	(-2.53)	-0.76	(-1.99)

This table present the returns, Fama and French three factor model alpha and Fama-French-Cahart four factor model alpha for portfolios sorted according to their Hellinger Tail Risk Beta (we consider a total of 1228 stocks, the constituents of the S&P 500 and Russel 1000 index, ranging from 01/02/2008 to 11/12/2015). The Hellinger Tail Risk betas are estimating using returns over the 252 trading days prior to portfolio formation. After portfolio assignment we track returns one day, one week (5 days) and one month (21 days) post formation. To facilitate comparison we convert daily returns and alphas into monthly percentages by multiplying them by 21 and 5 for daily and weekly holding periods respectively.T-statistics are calculated using Newey and West variance matrix.

Table 14: **Tail Risk Sorted Portfolios - Robustness**

Model	(1)		(2)		(3)		(4)	
	Ret.	t	Ret.	t	Ret.	t	Ret.	t
1	2.95	(2.72)	2.94	(2.64)	3.06	(3.02)	2.97	(2.78)
2	2.00	(2.25)	2.14	(2.38)	2.14	(2.68)	1.97	(2.34)
3	1.92	(2.42)	2.01	(2.48)	1.77	(2.42)	1.63	(2.18)
4	1.65	(2.26)	1.73	(2.31)	1.72	(2.49)	1.65	(2.36)
5	1.65	(2.43)	1.74	(2.54)	1.70	(2.55)	1.67	(2.54)
6	1.49	(2.35)	1.56	(2.42)	1.48	(2.25)	1.50	(2.40)
7	1.44	(2.42)	1.53	(2.57)	1.43	(2.28)	1.64	(2.73)
8	1.38	(2.43)	1.46	(2.66)	1.36	(2.22)	1.39	(2.32)
9	1.12	(2.12)	1.41	(2.77)	1.38	(2.20)	1.46	(2.43)
10	1.49	(2.56)	1.32	(2.61)	1.47	(2.15)	1.61	(2.29)
High - Low	-1.46	(-2.31)	-1.61	(-2.24)	-1.60	(-3.02)	-1.36	(-2.12)
FF3 Alpha	-1.47	(-2.29)	-1.62	(-2.27)	-1.19	(-2.97)	-0.95	(-1.81)
FF4 Alpha	-1.49	(-2.32)	-1.62	(-2.28)	-1.11	(-2.94)	-0.72	(-2.06)

This table present the returns, Fama and French three factor model alpha and Fama-French-Cahart four factor model alpha for portfolios sorted according to their Hellinger Tail Risk Beta (we consider a total of 1228 stocks, the constituents of the S&P 500 and Russel 1000 index, ranging from 01/02/2008 to 11/12/2015). The Hellinger Tail Risk betas are estimating using returns over the 252 trading days prior to portfolio formation. In Model (1) stocks are sorted according to their exposures to $TR - E[TR]$ where $E[TR]$ is estimated by fitting a AR(7) model (the "best" fitting time series model for the Hellinger Tail Risk), in Model (2) we adopt a more parsimonious AR(1) setting to calculate $E[TR]$, in Model (3) stocks are sorted based on their exposures to surprises in the Hellinger Tail Risk, defined as the periods were the Tail Risk excedes 1.65 standard deviations from its mean, in Model (4) stocks are sorting according to their exposure to the Hellinger Tail Risk calculated using one minute intra day data. After portfolio assignment we track returns one day, one week (5 days) and one month (21 days) post formation. To facilitate comparison we convert daily returns and alphas into monthly percentages by multiplying them by 21 and 5 for daily and weekly holding periods respectively.T-statistics are calculated using Newey and West variance matrix.

Table 15: **Tail Risk Double Sorted Portfolios**

Portfolio	1.00	2.00	3.00	4.00	5.00	High - Low	FF4
SIZE	2.63	1.95	1.70	1.40	1.08	-1.55	-0.66
	(2.64)	(2.52)	(2.52)	(2.43)	(2.08)	(-2.65)	(-2.00)
MOM	2.36	1.71	1.65	1.51	1.52	-0.84	-0.19
	(2.58)	(2.28)	(2.47)	(2.48)	(2.67)	(-1.94)	(-0.76)
D. Rev.	2.77	2.01	1.66	1.27	1.03	-1.74	-0.91
	(2.82)	(2.64)	(2.52)	(2.17)	(1.94)	(-3.15)	(-2.96)
W. Rev.	2.66	1.98	1.58	1.35	1.18	-1.48	-0.71
	(2.73)	(2.58)	(2.39)	(2.30)	(2.19)	(-2.73)	(-2.25)
M. Rev.	2.67	1.98	1.62	1.35	1.13	-1.55	-0.76
	(2.76)	(2.60)	(2.44)	(2.31)	(2.06)	(-2.93)	(-2.45)
ILLIQ.	2.60	1.96	1.64	1.42	1.13	-1.47	-0.61
	(2.66)	(2.52)	(2.46)	(2.45)	(2.16)	(-2.62)	(-1.89)
Turn.	2.76	2.16	1.70	1.65	1.58	-1.18	-0.47
	(2.89)	(2.77)	(2.50)	(2.69)	(2.77)	(-2.27)	(-1.36)
COSKEW	2.71	2.01	1.59	1.34	1.11	-1.60	-0.72
	(2.72)	(2.62)	(2.38)	(2.31)	(2.13)	(-2.78)	(-2.30)
IVOL	2.42	2.02	1.59	1.53	1.20	-1.22	-0.48
	(2.65)	(2.62)	(2.32)	(2.49)	(2.24)	(-2.61)	(-1.77)
MAX	2.46	1.95	1.64	1.46	1.24	-1.23	-0.57
	(2.76)	(2.58)	(2.40)	(2.35)	(2.21)	(-2.85)	(-2.08)
COKURT	2.73	1.90	1.62	1.36	1.13	-1.59	-0.73
	(2.74)	(2.47)	(2.46)	(2.34)	(2.18)	(-2.77)	(-2.29)
D. Beta	2.47	1.80	1.59	1.47	1.42	-1.05	-0.67
	(2.95)	(2.50)	(2.37)	(2.30)	(2.20)	(-3.24)	(-2.59
D. Vol	2.29	1.93	1.65	1.59	1.29	-1.00	-0.44
	(2.69)	(2.57)	(2.39)	(2.48)	(2.25)	(-2.68)	(-1.81)

This table present the returns, Fama and French three factor model alpha and Fama-French-Cahart four factor model alpha for double sorted portfolios. Each day we first sort stocks into five groups based on firms characteristics. Within each characteristics group we then sort stocks according to tail risk beta (we consider a total of 1228 stocks, the constituents of the S&P 500 and Russel 1000 index, ranging from 01/02/2008 to 11/12/2015) estimated using returns over the 252 trading days prior to portfolio formation. To form portfolios that are heterogeneous on the characteristics but homogeneous on tail risk exposure we compute the means on the beta sorted portfolios across characteristics according to their Hellinger Tail Risk Beta. For details on the sorting characteristics please see appendix A. For all models portfolios are re-balanced daily, the holding period equal one day, and the betas are estimated using a univariate regression. To facilitate comparison we convert daily returns and alphas into monthly percentages by multiplying them by 21. T-statistics are calculated using Newey and West variance matrix.

Table 16: **Tail Risk Reverse Double Sorted Portfolios**

Portfolio	1.00	2.00	3.00	4.00	5.00	High - Low	FF4
SIZE	2.39	1.79	1.73	1.50	1.33	-1.06	-0.93
	(2.97)	(2.49)	(2.53)	(2.21)	(2.13)	(-3.36)	(-6.34)
MOM	2.05	1.63	1.59	1.54	1.94	-0.11	0.03
	(2.44)	(2.33)	(2.44)	(2.38)	(2.72)	(-0.26)	(0.17)
D. Rev.	2.36	2.10	1.76	1.35	1.17	-1.19	-1.05
	(3.00)	(3.06)	(2.67)	(2.03)	(1.59)	(-3.43)	(-3.10)
W. Rev.	2.44	1.90	1.65	1.40	1.37	-1.07	-0.88
	(3.07)	(2.78)	(2.49)	(2.11)	(1.88)	(-3.31)	(-2.77)
M. Rev.	2.27	1.83	1.69	1.54	1.41	-0.86	-0.68
	(2.87)	(2.66)	(2.54)	(2.31)	(1.96)	(-2.45)	(-1.98)
ILLIQ.	1.38	1.50	1.64	1.84	2.39	1.02	0.84
	(2.26)	(2.27)	(2.33)	(2.50)	(2.98)	(3.13)	(5.32)
Turn.	2.06	1.96	1.94	1.98	1.89	-0.17	-0.43
	(2.94)	(2.86)	(2.76)	(2.78)	(2.40)	(-0.48)	(-1.29)
COSKEW	1.94	1.51	1.60	1.70	2.00	0.06	0.01
	(2.62)	(2.28)	(2.43)	(2.51)	(2.60)	(0.24)	(0.04)
IVOL	1.43	1.52	1.65	1.87	2.28	0.86	0.40
	(2.50)	(2.42)	(2.41)	(2.48)	(2.60)	(2.07)	(1.61)
MAX	1.53	1.57	1.64	1.66	2.35	0.83	0.29
	(2.81)	(2.50)	(2.39)	(2.16)	(2.64)	(1.84)	(1.16)
COKURT	1.97	1.68	1.57	1.69	1.85	-0.12	-0.12
	(2.61)	(2.51)	(2.39)	(2.51)	(2.44)	(-0.54)	(-0.57)
D. Beta	1.57	1.62	1.58	1.80	2.17	0.60	-0.09
	(2.97)	(2.65)	(2.30)	(2.37)	(2.36)	(1.26)	(-0.35)
D. Vol	1.34	1.48	1.65	1.77	2.51	1.18	0.54
	(2.53)	(2.43)	(2.38)	(2.31)	(2.71)	(2.34)	(1.93)

This table present the returns, Fama and French three factor model alpha and Fama-French-Cahart four factor model alpha for double sorted portfolios. Each day we first sort stocks into five groups based on the Hellinger Tail Risk beta, estimated using returns over the 252 trading days prior to portfolio formation. Within each beta group we then sort stocks according to firms characteristics (we consider a total of 1228 stocks, the constituents of the S&P 500 and Russel 1000 index, ranging from 01/02/2008 to 11/12/2015). To form portfolios that are heterogeneous on the tail risk beta but homogeneous on the characteristics we compute the means on the characteristics portfolios across beta groups. For details on the sorting characteristics please see appendix A. For all models portfolios are re-balanced daily, the holding period equal one day, and the betas are estimated using a univariate regression. To facilitate comparison we convert daily returns and alphas into monthly percentages by multiplying them by 21. T-statistics are calculated using Newey and West variance matrix.

Table 17: **Contemporaneous Downside Jump Regressions - Expected Shortfall**

	E. Shortfall	VIX	RV	EPU	ADS
	Panel A: Univariate Regressions				
β	0.10	0.04	0.07	0.03	-0.01
$t - stat.$	(5.21)	(3.55)	(3.41)	(2.62)	(-0.78)
R^2	0.14	0.03	0.07	0.01	0.00
	Panel B: Multivariate Regressions				
E. Shortfall		0.11	0.11	0.11	0.11
		(4.11)	(6.41)	(6.35)	(6.39)
VIX		-0.02	-0.03	-0.03	-0.01
		(-1.45)	(-1.07)	(-1.23)	(-0.34)
RV			0.00	0.00	0.00
			(0.13)	(0.13)	(0.08)
EPU				0.01	0.01
				(0.87)	(0.79)
ADS					0.03
					(1.82)
R^2		0.14	0.15	0.15	0.15

This table presents the results for contemporaneous regressions where the endogenous variable is the Downside Jump measure calculated in **??**. Panel A presents the results for univariate regressions where the explanatory variables are the Expected Shortfall, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruoba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). Panel B presents multivariate regressions where additional to the tail risk measure we also control for the above mentioned crash sensitive indexes. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 18: **Weekly Downside Jump Prediction Regressions - Segregated - Expected Shortfall**

	D. Jump 1	D. Jump 2	D. Jump 3	D. Jump 4	D. Jump 5	Aggregate
E. Shortfall	0.83	0.47	0.17	0.19	0.09	0.19
	(1.61)	(2.12)	(1.87)	(2.49)	(1.63)	(2.56)
VIX	-0.26	-0.31	-0.05	-0.03	-0.11	-0.17
	(-0.64)	(-1.82)	(-0.70)	(-0.47)	(-2.07)	(-2.59)
RV	-0.67	-0.30	-0.06	-0.08	-0.01	-0.13
	(-1.28)	(-1.44)	(-0.51)	(-0.88)	(-0.36)	(-1.84)
EPU	0.00	0.03	0.05	0.00	0.04	0.04
	(0.00)	(0.21)	(0.65)	(0.04)	(0.66)	(0.53)
ADS	-0.34	-0.17	0.04	0.00	-0.03	-0.10
	(-0.74)	(-0.95)	(0.55)	(-0.17)	(-0.45)	(-1.41)
Lag Jump	0.38	0.35	0.22	0.19	0.26	0.54
	(6.55)	(6.17)	(3.68)	(2.31)	(4.09)	(10.50)
Constant	5.88	1.34	0.43	0.20	0.24	0.54
	(9.12)	(8.93)	(7.38)	(6.68)	(7.25)	(8.75)
R^2	0.16	0.13	0.07	0.09	0.09	0.30
R^2*	0.01	0.01	0.01	0.05	0.02	0.03

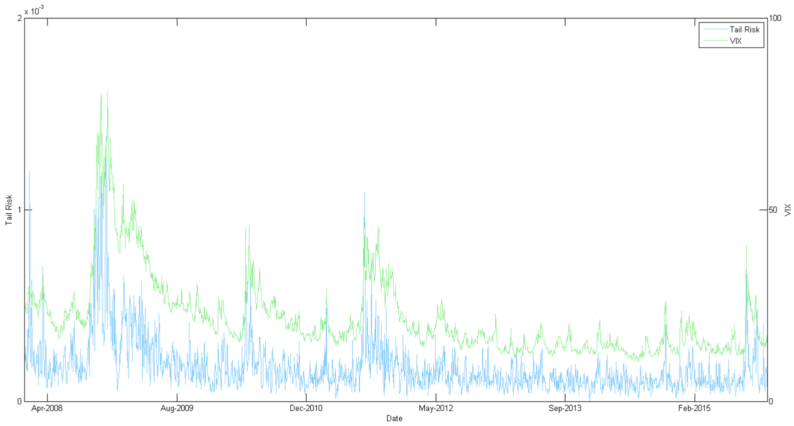
This table presents the results for weekly prediction regressions where the endogenous variable is the Downside Jump measure calculated in **??** and its five components. Results are from multivariate regressions where the explanatory variables are the Expected Shortfall, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruoba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). In the last line we also present the R^2* for univariate regressios where the only explanatory variable is the Expected Shortfall. t-statistics are calculated using Newey-West variance matrix with five lags.

Table 19: **Options Portfolios Prediction Regressions**

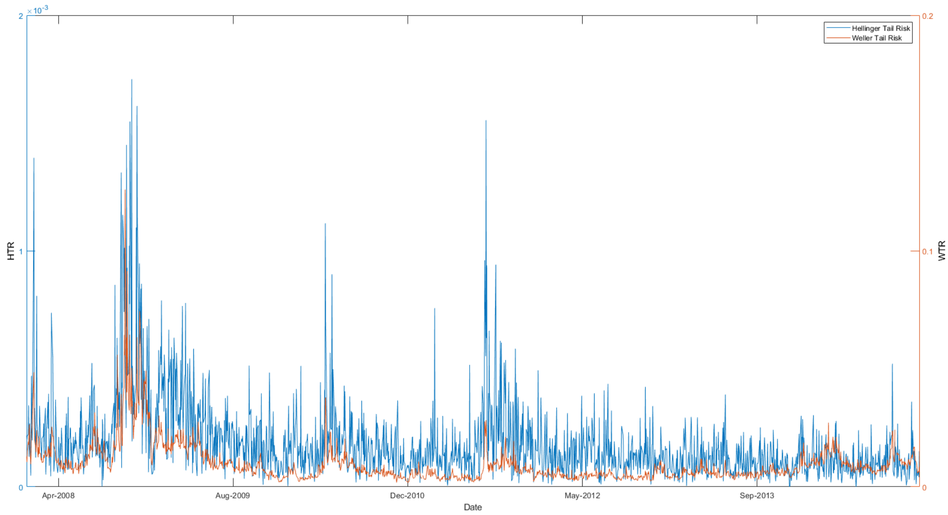
	DOTM	OTM	ATM	ITM	DITM	DOTM-DITM	Corr.
			Panel A: Contemporaneous				
E. Shortfall	0.29	0.30	0.28	0.18	0.12	0.17	
	(7.33)	(7.52)	(7.49)	(8.69)	(9.52)	(5.46)	
R^2	0.31	0.32	0.34	0.37	0.37	0.23	
R^2*	0.16	0.17	0.19	0.23	0.23	0.10	
			Panel B: Prediction				
E. Shortfall	0.06	0.06	0.05	0.04	0.03	0.04	
	(1.73)	(1.91)	(2.03)	(2.17)	(2.13)	(1.47)	
R^2	0.03	0.03	0.03	0.04	0.05	0.02	
R^2*	0.01	0.01	0.01	0.01	0.01	0.01	
			Panel C: Factor Model				
FF3 Alpha	-0.05	-0.03	0.03	0.04	0.02	-0.07	
	(-5.31)	(-3.99)	(3.87)	(11.33)	(10.72)	(-8.74)	-0.39
FF3+MOM Alpha	-0.05	-0.04	0.03	0.04	0.02	-0.07	
	(-5.40)	(-4.05)	(3.91)	(11.53)	(10.85)	(-8.85)	-0.43

This table presents the results for contemporaneous and one day ahead prediction regressions where the endogenous variables are portfolios formed on index options according to the options moneyness for put options with expiration dates between 1 and 45 days from the portfolio formation date. The first three lines present the estimated beta, t-statistics and R^2 from multivariate contemporaneous regressions where the explanatory variables are the Expected Shortfall, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruoba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016), the Downside Jump measure. The fourth line presents the estimated R^2 for a regression where the Expected Shortfall is the only explanatory variable. Lines 5-8 present the same statistics as lines 1-4 for prediction regressions where the lag of the portfolio return is a additional explanatory variable. Lines 9-12 present the alphas and t-statistics from Fama and French three factors model and Fama-French-Cahart four factor models respectively. The last column presents the correlation between alphas and multivariate regressions Expected Shortfall betas. t-statistics are calculated using Newey-West variance matrix with five lags.

VIX and Hellinger Tail Risk



Weller Tail Risk and Hellinger Tail Risk



Weller, Hellinger Moving Average Tail Risk and VIX

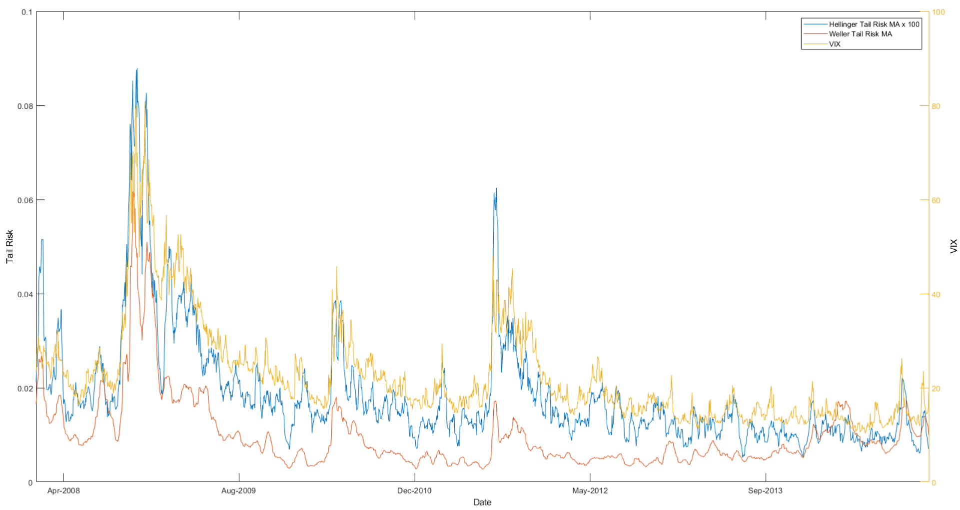


Figure 1: This figure plots the estimated, daily, Hellinger Tail Risk and the daily VIX index. This figure plots a 10 days moving average for the estimated Hellinger Tail Risk and the daily VIX index.

Industry Portfolios Betas

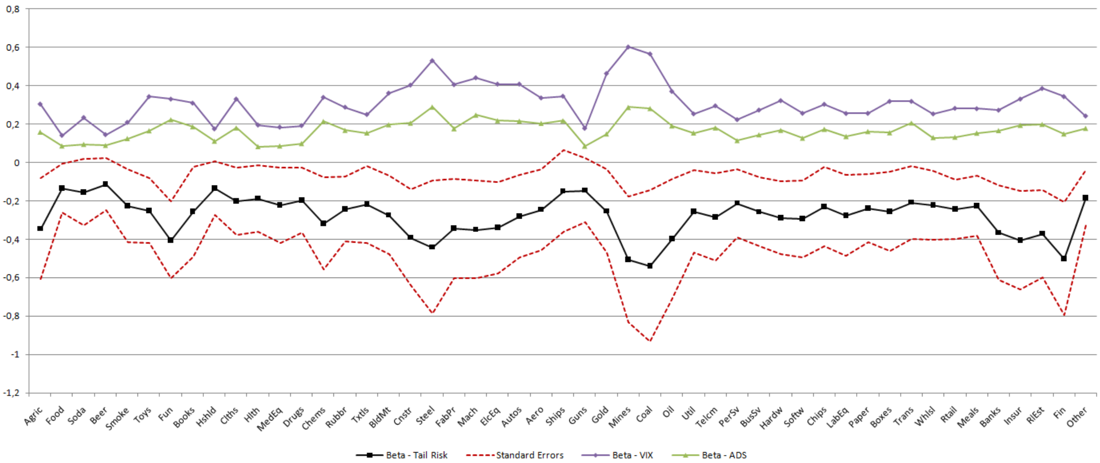


Figure 2: This figure plots the estimated betas for the Hellinger Tail Risk, CBOE VIX index and the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB for multivariate regressions where the endogenous variable are returns on the 49 Industry Portfolios from Kenneth French library. Results are from multivariate regressions where the explanatory variables are the Hellinger Tail Risk, the CBOE VIX index, the realized variance measured as the squared sum of intraday, 15 minutes interval, S&P 500 returns, the Aruba, Diebold and Scotti Business Conditions Index calculated by the Philadelphia FRB and the Economic Policy Uncertainty Index for the U.S. economy from Baker et al. (2016). Red dashed lines indicate the 10% confidence bands for the Hellinger Tail Risk beta calculated using Newey and West variance matrix with five lags.