
Nonparametric Tail Risk, Stock Returns, and the Macroeconomy

Caio Almeida¹, Kym Ardison¹, René Garcia², and Jose Vicente³

¹EPGE/FGV, Rio de Janeiro, ²Université de Montréal and Toulouse School of Economics and ³IBMEC Business School and Banco Central do Brasil

Address correspondence to Caio Almeida, FGV-EPGE, Praia de Botafogo, 190, Rio de Janeiro, Brazil, or e-mail: caio.almeida@fgv.br.

Received January 31, 2017; accepted February 7, 2017

Abstract

This paper introduces a new tail-risk measure based on the risk-neutral expected shortfall of a cross-section of stock returns. We propose a novel way to risk neutralize the returns without relying on option price information. Empirically, we illustrate our methodology by estimating a tail-risk measure over a long historical period based on a set of size and book-to-market portfolios. We find that a risk premium is associated with long-short strategies with portfolio sorts based on tail-risk sensitivities of individual securities. Our tail-risk index also provides meaningful information about future market returns and aggregate macroeconomic conditions. Results are robust to the cross-sectional information and other parameters selected to compute the tail-risk measure.

Key words: economic predictability, prediction of market returns, risk factor, risk-neutral probability, tail risk

JEL classification: G12, G13, G17

* We would like to thank David Veredas, Bruno Feunou, Ron Gallant, Cedric Okou, and Marcio Laurini for useful discussions, and also Tim Bollerslev, Andrew Patton, and participants at the 2015 CIREQ Montreal Econometrics Conference, 2015 Econometric Society World Congress, 2015 TSE Financial Econometrics Conference, the First International Workshop in Financial Econometrics in 2013 (Natal), 2014 SoFIE Conference (Toronto), 2014 SoFIE Harvard Summer School, 2014 LAMES (Sao Paulo), 2015 Brazilian Meeting of Finance, 2015 Brazilian Meeting of Econometrics, seminar participants at the financial econometrics seminar at Duke University and ORFE financial mathematics seminar at Princeton University. We also thank Viktor Todorov and Bryan Kelly for providing their data sets. We especially thank Osmani Guillen for his help in the early stages of this project. The first author acknowledges financial support from CNPq. The second author acknowledges financial support from ANBIMA. The third author is a research Fellow of CIRANO and CIREQ. The views expressed in the papers are the authors' and do not necessarily reflect those of the Central Bank of Brazil.

Following the 2007–2009 financial crisis, financial researchers as well as regulators devoted a lot of attention to the measure and analysis of tail risk and systemic risk and to the economic consequences of such risks. An important objective of this research agenda is to develop measures that are able to capture a common risk to many assets or institutions as opposed to individual risk measures such as Value-at-Risk (VaR) for one particular portfolio or firm. Although systemic risk focuses on the financial institutions, tail risk looks more generally at the risk provoked by catastrophic events or disasters and affects all firms. It could be measured with the time series of one firm or one portfolio but rare occurrences of disasters make this estimation at best very imprecise. Therefore, the most recent approach to measuring tail risk has been to exploit the richness of the cross-section of returns. By proposing a power law model for the left tail of asset returns, [Kelly and Jiang \(2014\)](#) obtain a new measure of time-varying tail risk that captures common fluctuations in tail risk among individual stocks. [Allen, Bali, and Tang \(2012\)](#) measure systemic risk through the 1% VaR of several tail distributions of the cross-section of returns of financial firms. The main additional advantage of these measures is the availability of long historical data for equity returns at the daily frequency. Both studies use the fact that tail risk plays an important role in explaining risk premia of equities and contains valuable information for predicting future economic conditions. One shortcoming is that this measure of tail risk is extracted directly from raw returns without risk neutralization, which is an adjustment of statistical returns for changes in risk aversion or time preferences or other changes in economic valuations.

Risk-neutralized measures of tail risk have used option prices. [Ait-Sahalia and Lo \(1998\)](#) and [Ait-Sahalia and Lo \(2000\)](#) introduced an option-based methodology to recover the risk-neutral densities for the S&P 500 index. In these papers, the authors stress that risk-neutral information coming from option prices provides an additional, economically relevant, source of information to estimate tail risk, in their case VaR. Recently, [Bollerslev and Todorov \(2011\)](#) proposed a model-free index of investors' fear adopting a database of intraday prices of futures and the cross-section of S&P 500 options. They show that tail-risk premia account for a large fraction of expected equity and variance risk premia as measured by the S&P 500 aggregate portfolio. In a complementary work, [Bollerslev, Todorov, and Xu \(2015\)](#) provide empirical evidence that the jump risk component of the variance risk premium is a strong predictor of future market returns. Still relying on S&P 500 index options, based on its empirical distribution, [Bali, Cakici, and Chabi-Yo \(2011\)](#) propose risk neutral and objective measures of riskiness, and show that they are good predictors of future market returns. [Siriwardane \(2013\)](#) proposes another interesting method to estimate the risk of extreme events. He extracts daily measures of market-wide disaster risk from a cross-section of equity option portfolios (in particular from puts) using a large number of firms, and show that these measures are useful to predict business cycle variables and to construct profitable portfolios sorted by disaster risk. A major limitation of these studies is the short time span for the availability of option and high-frequency data, in the order of 20 years, especially when the focus is on rare events. Moreover, several markets around the world do not have well-established and liquid option markets.

We propose an approach based on a cross-section of portfolio returns, together with a risk neutralization. Our aim is to develop a simple measure based on easily available data with a long history and applicable to a vast set of asset markets. This disqualifies option prices used in the previously cited studies. However, we want to reinterpret the insight provided in [Ait-Sahalia and Lo \(2000\)](#) by eliciting state-price densities with a nonparametric

methodology developed in [Almeida and Garcia \(2016\)](#). Based on a family of discrepancy functions, they derive nonparametric stochastic discount factor (SDF) bounds that naturally generalize variance ([Hansen and Jagannathan, 1991](#)), entropy ([Backus, Chernov, and Zin, 2014](#)), and higher-moment ([Snow, 1991](#)) bounds. Implicit in these derivations is the computation of the SDF itself that provides the risk neutralization. Therefore, our new tail-risk measure is based on the risk-neutral expected shortfall (ES) of a set of portfolio returns. Our aggregate tail-risk measure will simply be the average of the individual portfolio ESs.

To assess the empirical relevance of our tail-risk measure, we first look at how well it matches the main extreme market events and the business cycle during a long sample period (July 1926 to April 2014). It captures most events, is more volatile around these events, and appears as strongly counter-cyclical. We also verify that it correlates well with other measures of tail risk based on high-frequency and option-implied data as well as macroeconomic variables. Since our measure is based on risk-neutralized returns, it is essential to compare it with a measure using both returns and option data. We replicate the methodology used in [Ait-Sahalia and Lo \(1998, 2000\)](#) and [Ait-Sahalia and Lo \(2000\)](#) for the period January 1996–April 2014 and estimate a comparable tail-risk measure. The time series from our own tail-risk measure based only on the cross-section of returns follows closely their tail-risk measure time series. These basic properties are therefore reassuring in qualifying our measure as risk-neutral tail risk.

The second empirical test is to establish the existence and measure the magnitude of a tail-risk premium in equity returns. Our main hypothesis, as in [Kelly and Jiang \(2014\)](#), is that investors' marginal utility is increasing in tail risk. Therefore, we explore long-short portfolios based on tail-risk hedging capacity exposures and show that, even after controlling for several benchmark factors, this strategy provides a sizable negative and statistically significant alpha for both one-month and one-year horizons. The results are also very similar to the option-based methodology of [Ait-Sahalia and Lo \(1998\)](#) and [Ait-Sahalia and Lo \(2000\)](#) on the shorter sample.

Next, we provide evidence that our tail-risk measure anticipates stock market movements for intermediate horizons, even after controlling for several known predicting variables. Our measure has also some predictive power for economic conditions. We use the predictive regressions framework of [Allen, Bali, and Tang \(2012\)](#) with forecasting horizons varying from 1 to 12 months. Our proposed tail-risk measure reasonably anticipates future declines of the main macroeconomic condition indexes. To establish the “channels” through which tail risk affects the real economy we also consider [Bloom \(2009\)](#) vector auto-regressive framework. We show that short-term impulse responses to shocks in tail risk are associated with lower levels of employment and industrial production and are of greater magnitude than volatility shocks.

In a robustness section, we show how the risk premia and predictive properties vary with the proposed measure when we change such parameters as the number and nature of portfolios used to compute the excess ES, the window length, and the VaR threshold as well as the discrepancy parameter that controls the risk neutralization. Overall our conclusion is that the results are robust in general and most notably when we use different portfolios such as industry portfolios or financial and real sector portfolios.

[Brownlees and Engle \(2015\)](#) also adopted a shortfall measure to assess systemic risk but proposed a market-based measure of conditional capital shortfall for individual financial firms (exposed to capital regulation). We differ by considering a risk-neutral measure for a larger cross-section of returns. Of course, our measure could be specialized to financial

firms to consider systemic risk and not simply tail risk. Our paper is also intimately related to [Adrian and Brunnermeier \(2014\)](#) by its use of cross-sectional information. However, they also consider a systemic risk measure that captures financial industry co-movements or financial contagion. In our methodology, interconnectedness of assets returns comes from the risk-neutral probability that gives more weight to states of nature where assets in the chosen panel have lower returns. Finally, this paper also expands the analysis performed by [Bali, Demirtas, and Levy \(2009\)](#) in several dimensions. First, while their risk measures are based on objective probabilities, our tail risk exploits the important information in terms of downside risk embedded in risk neutral probabilities. Also, while [Bali, Demirtas, and Levy \(2009\)](#) focus mainly on portfolio formation according to downside risk exposures, our paper puts more emphasis on analyzing the macroeconomic implications of tail risk and the channels through which it affects the real economy.

The rest of the paper is organized as follows. Section 1 describes how we construct a risk-neutralized shortfall measure of tail risk based on a panel of asset returns and explains how it differs from other available tail-risk measures. We compute a long time series of such tail risk based on a set of size and book-to-market portfolios. Section 2 starts by analyzing the correlation of our tail-risk measure with the main existing tail and systemic risk measures and market index returns. We analyze in detail the relationship of the proposed measure with its option counterpart using [Ait-Sahalia and Lo \(2000\)](#) methodology and with the [Kelly and Jiang \(2014\)](#) measure. In Section 3, we study empirically the predictive properties of our tail-risk measure for market returns and macroeconomic activity indicators. Section 4 analyzes the robustness of our tail-risk measure in various dimensions. Section 5 concludes with a summary and some potential extensions.

1 A Nonparametric Tail-Risk Measure

1.1. Building a Risk-Neutral Excess ES Measure

Historical VaR was considered for a long time as a good tool for managing the risk of a portfolio. However, given the relative scarcity of extreme events in historical samples, several authors proposed to build measures based on risk-neutral probabilities assigned to historical observations. These were generally computed from option prices on the underlying (generally the market) as in [Ait-Sahalia and Lo \(2000\)](#) and several other papers.¹ These probabilities incorporate economic conditions and the risk attitude of investors and are therefore more reliable than historical probabilities. However, since prices for liquid options are not readily available for many assets and countries, a cross-sectional approach based on returns has been suggested to compute risk associated with tails.²

For both theoretical and empirical reasons, we choose to base our measure on excess ES instead of VaR.³ A main advantage of using threshold exceedances comes from the

- 1 See in particular [Ait-Sahalia and Lo \(1998\)](#), [Breedon and Litzenberger \(1978\)](#), [Bates \(1991\)](#), [Rubinstein \(1994\)](#), and [Longstaff \(1995\)](#).
- 2 The most recent literature includes [Kelly and Jiang \(2014\)](#); [Allen, Bali, and Tang \(2012\)](#); and [Adrian and Brunnermeier \(2014\)](#) to cite a few.
- 3 [Basak and Shapiro \(2001\)](#) provide an equilibrium analysis that puts forward various counter-intuitive implications of VaR management and show that ES avoids them. In addition, ES is a coherent measure of risk (see [Artzner et al., 1999](#)).

information it contains about the *whole* tail of the distribution instead of just a point-wise percentile as VaR.

For a particular asset i , we define the excess ES as follows:

$$\text{TR}_{i,t} = E^{\mathcal{Q}(R)}[(R_{i,\tau} - \text{VaR}_\alpha(R_{i,\tau})) | (R_{i,\tau} \leq \text{VaR}_\alpha(R_{i,\tau}))], \quad (1)$$

where t is the time period for which we are calculating the tail risk, τ denotes the possible states of nature, α is the VaR threshold, and $\mathcal{Q}(R)$ indicates the risk-neutral density. Our aggregate market tail-risk measure will be based on the average of the ES $\text{TR}_{i,t}$ of several portfolios i . Note that in this design we model \mathcal{Q} as a function of the observable returns. Throughout the paper we calculate tail risk at a monthly frequency. Thus, as it will become clearer further on, the states of nature (τ) will be captured by a number of past daily returns. By keeping this number relatively small our risk measure will react more quickly to changes in market conditions given that recent cross-sectional values of asset returns will be used to estimate the risk-neutral density. This is in contrast with usual VaR measures based on statistical historical properties of asset returns.⁴

1.2. A Non-Parametric Risk-Neutral Density

To circumvent the problems related to the availability of options prices, we propose to calculate the RND via nonparametric methods. To that end, Hansen and Jagannathan (1991) seminal paper proposes to minimize a quadratic loss function to estimate a SDF that prices exactly a set of basis assets. This insightful approach has been used extensively to assess the adequacy of financial models with a mean–variance SDF frontier. Almeida and Garcia (2016) have generalized their methodology to frontiers that involve all moments of the return distributions and use them to gauge models involving tail events such as disaster and disappointment aversion models. Given a series of assets returns, in an incomplete market where there are more states of nature than assets, they find a family of SDFs that minimize convex functions defined in the space of admissible and strictly positive SDFs. These convex functions measure the distance between an admissible SDF and the constant SDF of a risk-neutral economy. Assuming a constant short-term rate and homogenous physical probabilities, just as in a VaR historical simulation, we are able to obtain a direct correspondence between SDFs and RNDs.

Given that RND are the building blocks of our tail-risk procedure we succinctly expose the methodology adopted and its implications. Let (Ω, \mathcal{F}, P) be a probability space, and R denote a K -dimensional random vector on this space representing the returns of K primitive basis assets. In this static setting, an admissible SDF is a random variable m for which $E(mR)$ is finite and satisfies the Euler equation:

$$E(mR) = 1_K, \quad (2)$$

where 1_K represents a K -dimensional vector of ones.

As in Hansen and Jagannathan (1991), Almeida and Garcia (2016) are interested in the implications of Equation (2) for the set of existing SDFs. For a sequence of (m_t, R_t) that satisfy Equation (2) for all t , and observing a time series $\{R_t\}_{t=1, \dots, T}$ of basis assets returns, we

4 For an interesting exception, see Glasserman (2015) for a data-driven selection of historical stress test scenarios that are both extreme and plausible based on empirical-likelihood estimators.

assume that the composite process (m_t, R_t) is sufficiently regular such that a time series version of the law of large numbers applies.⁵ Therefore, sample moments formed by finite records of measurable functions of data R_t will converge to population counterparts as the sample size T becomes large.

Given a sample of basis assets returns, the set of admissible SDFs will depend on the market structure. The usual case is to have an incomplete market, that is, the number of states of nature (T) larger than the number of basis assets K . In such case, an infinity of admissible SDFs will exist, and if there is no in-sample arbitrage on the basis assets payoff space (see [Gospodinov, Kan, and Robotti, 2016](#)), there will exist at least one strictly positive SDF (see [Duffie, 1996](#)). For each strictly positive SDF there will be a corresponding risk-neutral density.

Given a convex discrepancy (penalty) function ϕ the generalized, in sample, minimum discrepancy problem proposed by [Almeida and Garcia \(2016\)](#) can be stated as:

$$\begin{aligned} \hat{m}_{\text{MD}} &= \arg \min_{\{m_1, \dots, m_T\}} \frac{1}{T} \sum_{i=1}^T \phi(m_i) \\ \text{subject to } & \frac{1}{T} \sum_{i=1}^T m_i (R_i - \frac{1}{a} \mathbf{1}_K) = \mathbf{0}_K \\ & \frac{1}{T} \sum_{i=1}^T m_i = a \\ & m_i \geq 0 \text{ (or } m_i > 0) \forall i \end{aligned} \quad (3)$$

In this optimization problem, restrictions to the space of admissible SDFs come directly from the general discrepancy function ϕ . The conditions $E(m(R - \frac{1}{a} \mathbf{1}_K)) = \mathbf{0}_K$ and $E(m) = a$ must be obeyed by any admissible SDF m with mean a . In addition, whenever there is a strictly positive solution the implied minimum discrepancy SDF is compatible with absence of arbitrage in an extended economy that considers derivatives over the underlying basis assets.⁶ The choice to impose a non-negativity or strict positivity constraint in the optimization problem is dictated by the choice of the discrepancy function $\phi(\cdot)$ [see [Almeida and Garcia \(2016\)](#) for a detailed analysis].

Despite the straightforward interpretation of the problem in [Equation \(3\)](#), its solution is not easy given that the number of unknowns is as large as the size of the sample. Therefore, [Almeida and Garcia \(2016\)](#) show that one can solve an analogous simpler dual problem:

$$\hat{\lambda} = \arg \sup_{\alpha \in \mathbb{R}, \lambda \in \Lambda} a * \alpha + \frac{1}{T} \sum_{i=1}^T \phi^{*+} \left(\alpha + \lambda' \left(R_i - \frac{1}{a} \mathbf{1}_K \right) \right), \quad (4)$$

- 5 For instance, stationarity and ergodicity of the process (m_t, R_t) are sufficient (see [Hansen and Richards, 1987](#)). In addition, we further assume that all moments of returns R are finite in order to deal with general entropic measures of distance between pairs of SDFs.
- 6 It is important to note that the homogeneous probability assumption will not affect the key insights we derive from this methodology and, if desired, one could also consider a kernel density to model the physical probabilities without additional complications.

where $\Lambda \subseteq \mathbb{R}^K$ and $\phi^{*,+}$ denote the convex conjugate of ϕ restricted to the non-negative (or strictly positive) real line,

$$R_i - \frac{1}{a} \mathbf{1}_K \phi^{*,+} = \sup_{w \in [0, \infty) \cap \text{domain } \phi} zw - \phi(w). \quad (5)$$

In this dual problem, λ can be interpreted as a vector of K Lagrange multipliers that comes from the Euler equations for the primitive basis assets in Equation (3). Almeida and Garcia (2016) specialize the ϕ function to the Cressie-Read family, $\phi^\gamma(m) = \frac{m^{\gamma+1} - a^{\gamma+1}}{\gamma(\gamma+1)}$, $\gamma \in \mathbb{R}$. This family captures as particular cases minimizations of variance (Hansen and Jagannathan, 1991), higher moments (Snow, 1991), and different kinds of entropy (Stutzer, 1996) estimators. With this family, closed-form formulas are obtained for λ and \hat{m}_{MD} :

i. if $\gamma > 0$,

$$\hat{\lambda}_\gamma = \arg \sup_{\lambda \in \Lambda_{\text{CR}}} \frac{1}{T} \sum_{i=1}^T \left(\frac{a^{\gamma+1}}{\gamma+1} - \frac{1}{\gamma+1} \left(a^\gamma + \gamma \lambda' \left(R_i - \frac{1}{a} \mathbf{1}_K \right) \right)^{\frac{\gamma+1}{\gamma}} \right) I_{\Lambda_{\text{CR}}(\mathbf{R})}(\lambda), \quad (6)$$

where $\Lambda_{\text{CR}} = \{ \lambda \in \mathbb{R}^K \mid \forall i = 1, \dots, T (a^\gamma + \gamma \lambda' (R_i - \frac{1}{a} \mathbf{1}_K)) > 0 \}$, $I_A(x) = 1$, if $x \in A$, and 0 otherwise, and:

$$\hat{m}_{\text{MD}}^i = a \frac{\left(a^\gamma + \gamma \hat{\lambda}'_\gamma (R_i - \frac{1}{a} \mathbf{1}_K) \right)^{\frac{1}{\gamma}}}{\frac{1}{T} \sum_{i=1}^T \left(a^\gamma + \gamma \hat{\lambda}'_\gamma (R_i - \frac{1}{a} \mathbf{1}_K) \right)^{\frac{1}{\gamma}}} I_{\Lambda_{\text{CR}}(\mathbf{R})}(\hat{\lambda}_\gamma); \quad (7)$$

ii. if $\gamma \leq 0$,

$$\hat{\lambda}_\gamma = \arg \sup_{\lambda \in \Lambda_{\text{CR}}} \frac{1}{T} \sum_{i=1}^T \left(\frac{a^{\gamma+1}}{\gamma+1} - \frac{1}{\gamma+1} \left(a^\gamma + \gamma \lambda' \left(R_i - \frac{1}{a} \mathbf{1}_K \right) \right)^{\frac{\gamma+1}{\gamma}} \right), \quad (8)$$

and:

$$\hat{m}_{\text{MD}}^i = a \frac{\left(a^\gamma + \gamma \hat{\lambda}'_\gamma (R_i - \frac{1}{a} \mathbf{1}_K) \right)^{\frac{1}{\gamma}}}{\frac{1}{T} \sum_{i=1}^T \left(a^\gamma + \gamma \hat{\lambda}'_\gamma (R_i - \frac{1}{a} \mathbf{1}_K) \right)^{\frac{1}{\gamma}}}. \quad (9)$$

The implied SDF depends directly on the parameter γ . Different choices of γ will weight differently higher-order moments of returns (see Almeida and Garcia, 2016). The choice of γ and the robustness of results with respect to its value are therefore important elements of our empirical strategy. We discuss these aspects in Sections 1.4 and 4, respectively.

To obtain the risk-neutral probabilities associated with each observation interpreted as a state of nature, we distort the usual $1/T$ measure by the computed SDF in Equation (9) adjusted by the interest rate:

$$\pi_i^{\text{RN}} = \frac{m_i(1+r)}{T}. \quad (10)$$

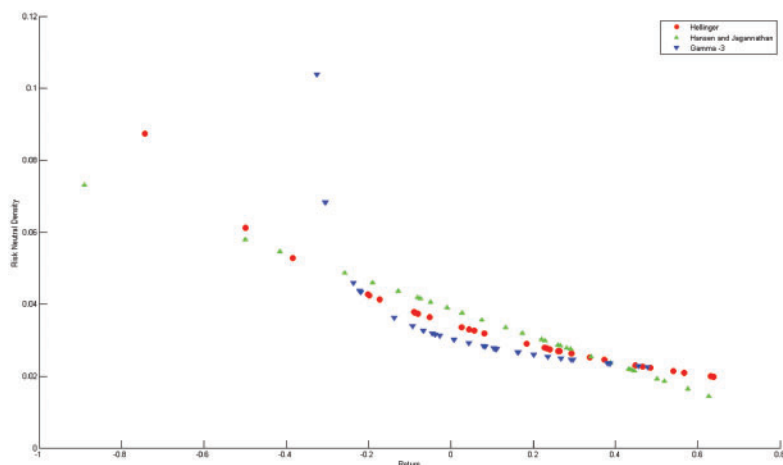


Figure 1. This figure presents the implied risk-neutral probabilities for a five asset economy and a variety of γ 's.

We illustrate the distortions in [Figure 1](#) for a five-asset economy that we will make explicit in the next section. On the horizontal axis, we plot the returns on the optimal portfolio resulting from the optimization in [Equation \(8\)](#), while the risk-neutral probabilities are featured on the vertical axis for various values of the parameter γ . The different lines illustrate clearly the fact that for more negative values of γ the implied risk-neutral probabilities give more weight to low returns or “bad” states of nature from an investor perspective. From the linear outcome generated by the unit value ([Hansen and Jagannathan, 1991](#)), the convexity increases as γ becomes more negative.

1.3. Choosing the Basis Assets and the Estimation Window

As mentioned earlier in this section, our risk-neutral density estimation methodology assumes an incomplete market framework. Empirically this translates into an upper bound in the number of cross-sectional assets for a given number of states of nature T . More precisely, for a given number of days (states of nature) T we must consider a number of assets K such that $K < T$. Most papers ([Kelly and Jiang, 2014](#), in particular) compute a tail-risk measure at a monthly frequency. To make our measure conditional on recent information, we therefore limit our sample to the former 30 days preceding the last day of each month.⁷ Otherwise we would be using old information to predict current and future stock market and economic conditions. Therefore, we cannot adopt a strategy similar to [Kelly and Jiang \(2014\)](#) who include all securities available in the CRSP data base at the end of each month and pool all their returns in the former 30 days to compute their tail-risk measure based on the Hill estimator.⁸

7 In the robustness section of the paper, Section 4 we report the sensitivity of our empirical results to the panel length.

8 In [Almeida, Ardison, and Garcia \(2016\)](#), we compute a tail-risk measure using a similar methodology with individual assets in a high-frequency environment.

Table 1. Principal component variance

Principal component	1	2	3	4	5
Variance (cumulative)	0.62	0.76	0.83	0.86	0.89

Notes: The cumulative variance of the first five principal components is reported. The principal component analysis was performed for the whole sample (07/1926–04/2014). Each month we compute the five principal components using the daily returns of the last 30 days for the 25 size and book-to-market Fama–French portfolios, and the corresponding cumulative explained variances. The reported figures are the averages of the monthly figures over the whole sample.

Given an upper bound of 30 days, we need to base our measure on portfolios. But we can ask ourselves whether we should limit ourselves to the market portfolio or exploit the cross-sectional information in say the usual 25 size and book-to-market portfolios.⁹ Bali, Brown, and Caglayan (2014) argue that previous studies have shown that investors usually hold portfolios composed by some market proxy (e.g., stock funds) and individual stocks. In this environment, individual stocks play a crucial role on the total portfolio tail risk. Empirically, they find evidence that market tail risk is not capable of providing useful information for return prediction whereas individual assets tail-risk measures (idiosyncratic tail risk) is quite successful in predicting market returns. Expanding on this, Kelly and Jiang (2014) also argues that the whole cross-section of stocks contains valuable information regarding the tail behavior of the returns. Therefore, it seems that we should rely on as much cross-sectional information as possible.

However, using the whole set of 25 portfolios as basis assets in our procedure might produce risk-neutral probabilities that result in sizable pricing errors on those basis assets.¹⁰ Recently Kozak, Nagel, and Santosh (2015) show that a SDF based on the first few principal components of asset returns explains many anomalies. Their argument rests on the fact that since asset returns have substantial commonality, absence of near-arbitrage opportunities implies that the SDF depends only on a few sources of return variation. Therefore, instead of considering the whole cross-section of the 25 size and book-to-market portfolios we extract the first five principal components from the 30-day panel.¹¹ To make our measure conditional, we compute the principal components for every end-of-month in the sample. Thus, for each t , we are not using future information in the tail-risk calculation. Table 1 reveals that these principal components are responsible for much of the portfolio variations. With five principal components, close to 90% of the variation is explained for the full sample.

1.4. The Choice of γ

To understand the effect of γ on our tail-risk measure, we derive a Taylor expansion of the expected value of $\phi(m) = \frac{m^{\gamma+1} - a^{\gamma+1}}{(\gamma(\gamma+1))}$ around the SDF mean a . Noting that $\phi(a) = 0$, $\phi'(m) =$

9 For more details about these portfolios, see <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html> (accessed 17 February 2017).

10 We also computed our tail-risk measure for the whole set of 25 Fama–French portfolios. While the pricing errors were higher, the overall empirical findings were preserved.

11 In our thorough comparison with the Kelly–Jiang measure in Section 2.3, we also compute our measure with principal components computed from the whole cross-section of returns.

$\frac{m^\gamma}{\gamma}$, $\phi''(m) = m^{\gamma-1}$, $\phi'''(m) = (\gamma - 1) m^{\gamma-2}$, $\phi^{(4)}(m) = (\gamma - 1)(\gamma - 2)m^{\gamma-3} \dots$, Taylor expanding ϕ and taking expectations on both sides we obtain¹²:

$$E(\phi(m)) = \frac{a^{\gamma-1}}{2} E(m - a)^2 + \frac{(\gamma - 1)a^{\gamma-2}}{3!} E(m - a)^3 + \frac{(\gamma - 1)(\gamma - 2)a^{\gamma-3}}{4!} E(m - a)^4 + \dots \tag{11}$$

Two important effects appear regarding the weights attributed to skewness and kurtosis in this expansion. First, for values of γ close to one, both skewness and kurtosis have small weights when compared with the variance that has a weight equal to one half in the expansion. This implies that discrepancies with values of γ close to one do not capture much of the higher moment activity of pricing kernels. Once we move to more negative values of γ both skewness and kurtosis receive considerable weights in the expansion. The second important aspect refers to the relative weights that are given to skewness and kurtosis by different Cressie-Read functions. For $-2 < \gamma < 1$ note that the absolute weight given to kurtosis is smaller than the corresponding weight given to skewness. However, for values of $\gamma < -2$, kurtosis receives more weight than skewness. In fact all even higher moments receive more absolute weight than their corresponding odd higher moments in this region of γ .

To choose a base-case value of γ , we rely on a comparison based on a disaster model. In Almeida and Garcia (2016), we derive bounds for the disaster model in Backus, Chernov, and Zin (2014). In this model the logarithm of consumption growth is given by:

$$g_{t+1} = \eta_{t+1} + J_{t+1}, \tag{12}$$

where η_{t+1} is the normal component $\mathcal{N}(\mu, \sigma^2)$ and J_{t+1} is a Poisson mixture of normals. The number-of-jumps variable j takes integer values with probabilities $e^{-\tau} \frac{\tau^j}{j!}$, where τ is the jump intensity. Conditionally on the number of jumps, J_t is normal:

$$J_t | j \sim \mathcal{N}(j\alpha, j\lambda^2). \tag{13}$$

In this model, the logarithm of the SDF with power utility is:

$$\log m_{t+1} = \log \beta - \zeta g_{t+1}, \tag{14}$$

where ζ is the coefficient of relative risk aversion. Therefore, the mean of the SDF is:

$$a = \exp \left\{ \log \beta - \zeta \mu + \frac{1}{2} (\zeta \sigma)^2 + \tau (e^{-\zeta \alpha + 0.5 (\zeta \lambda)^2} - 1) \right\}. \tag{15}$$

The discrepancy bound for the Cressie-Read family is the expectation of $\phi^\gamma(m)$. It can easily be obtained by taking the expectation of $\exp(\gamma + 1) \log m$, that is:

$$d = \exp(\gamma + 1) \log(\beta) - \zeta \mu (\gamma + 1) + 0.5 * (\zeta \sigma (\gamma + 1))^2 + \tau (\exp(-\zeta(\gamma + 1)\alpha + 0.5(\zeta(\gamma + 1)\lambda)^2) - 1). \tag{16}$$

12 Note that all functions in the Cressie-Read family are analytic, that is, their derivative of any order exists. For this reason, the only condition that is needed for the Taylor expansion to be valid is the existence of the first four moments of the MD SDF.

These bounds have a direct link with the measure of entropy used in [Backus, Chernov, and Zin \(2014\)](#). When $\gamma = -1$, our discrepancy function is $\log(a) - E(\log(m))$, which corresponds precisely to the entropy of the pricing kernel reported in their [Equation \(13\)](#).

For these bounds we are computing the Cressie-Read bounds with the returns on the S&P 500 index and equity options strategies on this index. We use four options portfolios that consist of highly liquid at-the-money (ATM) and out-of-the-money (OTM) European call and put options on the S&P 500 composite index trading on the Chicago Mercantile Exchange.¹³ We want to illustrate here how the frontiers and the disaster model discrepancy vary with the values of γ . For the disaster model we use the parameter values reported in [Table II of Backus, Chernov, and Zin \(2014\)](#), except for the risk aversion parameter that we set at 8 to match a slightly higher equity premium. For the values of γ , we chose three values that will illustrate the relation between the bounds and the model. In [Figure 2](#), we plot the three frontiers corresponding to the three values of γ 0, -0.5 and -1 . We also plot in the corresponding colors the points corresponding to the discrepancy of the model at the mean SDF value 0.9946. For $\gamma = 0$ the model passes quite easily because the weights put on higher moments for building the frontier are relatively small. On the contrary, for $\gamma = -1$ the restrictions in the bound are more stringent and the model is far below the frontier. For the intermediate value of $\gamma = -0.5$, the model point is above the frontier but still closer than for the two other values.¹⁴ Given this illustrative exercise we will choose $\gamma = -0.5$ as our base case.

There are also some statistical arguments for choosing this class of loss functions [Kitamura, Otsu, and Evdokimov \(2013\)](#) showed that, under some technical assumptions, the Hellinger ($\gamma = -0.5$) is the more robust one based on an asymptotic perturbation criterion.¹⁵

Furthermore, if we want a tail-risk measure that incorporates higher-order moments information we must compute a SDF (and therefore a RND) that also considers this information, in contrast to the Hansen and Jagannathan one. For instance, [Schneider and Trojani \(2015\)](#) argue that investors' "fear" is an aversion to downside risk, exactly what we aim at capturing with our methodology. Furthermore, they link the concept of "fear" with investor prudence,¹⁶ which is intimately related to negative skewness aversion. Therefore, a complete and informative risk measure should be able to reflect the informational content on higher-order moments.

Therefore for the majority of the empirical section we adopt the Hellinger Tail Risk as the main benchmark ($\gamma = -0.5$), but we will still present robustness tests with different values of γ in [Section 4](#).

¹³ These have been constructed by [Agarwal and Naik \(2004\)](#) to study performance of hedge funds.

¹⁴ We can use values close to -0.5 where the model point is even closer to the bound but one has to be careful because important discontinuities may suddenly occur and the frontier may get closer to the bound for $\gamma = -1$.

¹⁵ In a recent paper, exploring the non-linearity implied by the Hellinger estimator, [Schneider and Trojani \(2015\)](#) introduce a class of trading strategies that might be used to trade higher moments of the returns.

¹⁶ Almeida and Garcia ([2012, 2016](#)) demonstrated that the methodology we consider to estimate the SDF is compatible with this concept.

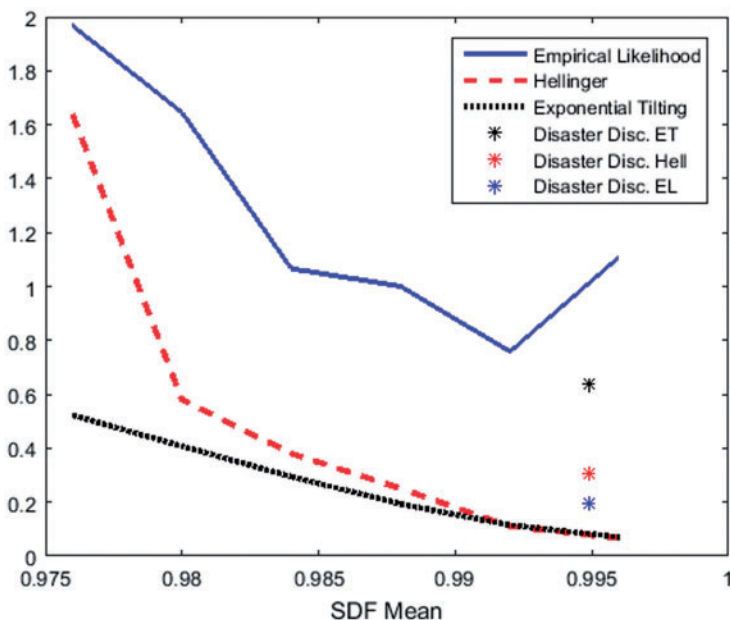


Figure 2. This figure plots discrepancy bounds for three Cressie-Read measures with the corresponding disaster-model coordinates. The basis assets are the returns on the S&P 500 index and equity options strategies on this index.

1.5. A First Look at the Tail-Risk Measure over Time

Figure 2 plots the evolution of our tail-risk measure, in blue, and a Hodrick–Prescott filtered version, in red, from July 1926 to April 2014. Our measure is very volatile and features various peaks often coincident with extreme or significant financial, economic, or political events. These events have affected negatively equity returns in important ways. Our tail-risk measure captures most of these huge market drops in U.S. history and also appears to be more volatile in the periods before and after some of them. Of course two economic and financial crisis periods figure prominently, the Wall Street Crash of 1929 followed by the Great Depression of 1929–1939 on the left, the beginning of our sample, and the recent financial crisis of 2008–2010 on the right of the graph. In between these two major crises, the filtered series shows that the overall level of the tail risk remained below these extremes, except for occasional spikes that are triggered by some specific events. For these sporadic events the mean reversion is much stronger, as in the dot-com bubble of the early 2000s or the recent European debt crisis starting at the end of 2009.

Other tail-risk measures have also captured these types of events but since many of them rely on option prices their span is much narrower, their sample starting at the beginning of the 1990s.

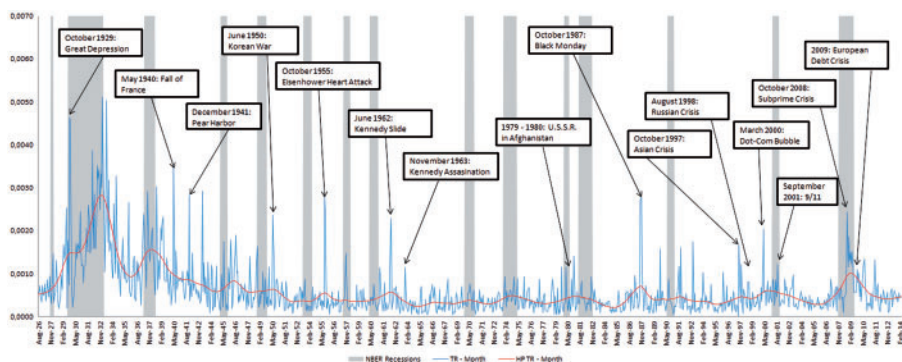


Figure 3. This figure plots the tail risk calculated using five principal components extracted from the 25 Fama and French size and book-to-market portfolios for the Hellinger estimated risk-neutral density ($\gamma = -0.5$). Principal components were extracted for the last day of each month in our sample using 30 days of past returns on the portfolios.

2 A Comparison With Other Tail-Risk Measures: Correlations and Risk Premia

Since many tail risk or downside risk measures have been proposed in the last 5 years, we start by computing simple correlations with several of them over different sample periods, some based on equity returns starting in the 60s, others using options computed from the beginning of the 90s. We also consider correlations with market indexes and macroeconomic indicators. We investigate in more depth the relationship of our measure with two prominent tail-risk measures, one based on option prices using the methodology of [Ait-Sahalia and Lo \(2000\)](#) and one recently proposed by [Kelly and Jiang \(2014\)](#) that uses the information in the whole panel of individual stock returns. In this section, we will conduct the comparison in terms of risk premia associated with portfolios sorted according to the hedging capacity of individual securities against tail risk. We will extend the comparison in Section 3 with the predictive properties of the measures for market returns and macroeconomic activity indicators.

2.1. Correlations with Other Indexes

In [Table 2](#), we report the correlation coefficients between our tail-risk measure and other tail-risk measures as well as financial and macroeconomic indexes: the tail-risk measure of [Bollerslev, Todorov, and Xu \(2015\)](#) based on option prices; the [Kelly and Jiang \(2014\)](#) tail-risk measure; the VIX; two stock market indexes; the S&P 500 and the CRSP portfolio; the systemic risk measure of [Allen, Bali, and Tang \(2012\)](#); and the macroeconomic conditions index of [Bali, Brown, and Caglayan \(2014\)](#).

The first interesting result is that our tail-risk measure has a noticeable correlation (0.43) with [Bollerslev, Todorov, and Xu \(2015\)](#) tail-risk measure despite the fact that we do not include any option returns in the computation of our measure. Another important measure based on option prices is the VIX. Its correlation with our tail-risk measure is 0.56. This measure is often interpreted as a good indicator for investors' crash fears. More generally, since option prices reflect investors' risk attitudes and economic conditions, it seems that our risk neutralization goes some distance in capturing this important

Table 2. Correlations with other tail-risk measures and financial and macroeconomic indicators

	Hellinger	S&P 500	CRSP	Bloom	KJ	BTX	VIX	Macro
Hellinger	1.0000							
S&P 500	-0.3210	1.0000						
CRSP	-0.2433	0.9842	1.0000					
Bloom	0.4572	-0.1333	-0.1498	1.0000				
KJ	-0.0723	0.0854	0.0802	-0.0202	1.0000			
BTX	0.4303	-0.1293	-0.1341	0.3726	-0.2289	1.0000		
VIX	0.5581	-0.3723	-0.3709	0.9288	-0.3820	0.6625	1.0000	
Macro	0.4684	-0.0578	-0.0243	0.5809	-0.2210	0.4395	0.5548	1.0000
CATFIN	0.4507	-0.4146	-0.4385	0.3811	-0.0819	0.5206	0.6463	0.5422

Notes: The table reports the correlation coefficients between Hellinger Tail Risk and other tail measures. CRSP denotes value-weighted CRSP stock index (07/1926–04/2014); Bloom refers to Bloom (2009) volatility factor (07/1962–06/2008); KJ refers to Kelly and Jiang (2014) tail-risk index (01/1963–12/2010); BTX refers to Bollerslev, Todorov, and Xu (2015) tail-risk index (01/19963–08/2013); VIX denotes the CBOE volatility index (01/1990–04/2014); Macro refers to Bali, Brown, and Caglayan (2014) macroeconomic uncertainty index (01/1994–12/2013); and CATFIN denotes Allen, Bali, and Tang (2012) systemic risk measure (01/1973–12/2012).

information for extreme risk. In Section 2.2, we will conduct a thorough comparison between a nonparametric measure incorporating option prices and our measure based only on risk-neutralized cross-sectional portfolio returns.

Another tail-risk measure has been proposed recently by Kelly and Jiang (2014), thereafter KJ tail-risk measure. Surprisingly, it is not at all correlated with our tail-risk measure. From a time series point of view, our measure is volatile and has little persistence, while the KJ measure is highly persistent. The nature of these measures is therefore very different. The source of this difference may be due to the amount of information used in constructing the two measures (the whole cross-section of returns for KJ, principal components of the 25 size and book-to-market portfolios in our measure), the fact that KJ is based on raw returns while ours is computed with risk-neutralized returns, or other differences regarding the VaR threshold and the estimating window. We will explore all aspects in a thorough comparison in Section 2.3. It should be noted that the KJ measure has a negative correlation of -0.38 with the VIX and a small and positive correlation of 0.23 with the Bollerslev, Todorov, and Xu (2015) measure of tail risk.

As it should be expected, our measure is counter-cyclical, with negative correlations of -0.32 and -0.24 with the S&P 500 and CRSP returns, respectively. In Section 3.1, we will look more closely at the relation between tail risk and the future aggregate returns. Another important benchmark is the measure of systemic risk (CATFIN) based on the financial sector by Allen, Bali, and Tang (2012). The computation of CATFIN is related to our measure since it is estimated using both VaR and ES methodologies, with nonparametric and parametric specifications. The correlation with our tail-risk measure is 0.45 , which is reassuring since returns in the financial sector capture financial crisis risk which underlies a good portion of the movements in a more general tail-risk measure based on all equity returns. Allen, Bali, and Tang (2012) show that high levels of systemic risk in the banking sector

impact macroeconomic conditions. Similarly our tail-risk measure is correlated with macroeconomic indicators of Bloom (2009) and Bali, Brown, and Caglayan (2014) (0.46 and 0.47, respectively). In Section 3.2, we study in detail its relationship with the real economy through the uncertainty channel. The KJ measure of tail risk exhibits no counter-cyclical behavior nor does it relate to the macroeconomic indicators of Bloom (2009) and Bali, Brown, and Caglayan (2014).

These correlations suggest a common component between our tail-risk measure based on the cross-section of risk-adjusted returns and option-based measures of tails risk measures. They also point out the distinctive behavior of the KJ tail-risk measure despite the use of the cross-section of returns. We will probe further the relation between these two sets of measures in the next two sections.

2.2. A Detailed Comparison with an Option-Based Tail-Risk Measure

We have seen that our tail-risk measure is reasonably well correlated with several tail risk and systemic risk measures. Among them, we want to probe further the relation with option-based measures. Since we claim that our procedure produces a risk-neutral measure of tail risk even though we are using only equity returns, it would be reassuring to show that the extracted measure has much in common with a measure incorporating option prices. Our empirical strategy consists in applying the methodology in Ait-Sahalia and Lo (2000) to estimate the excess ES in Equation (1) using the S&P 500 index options and returns. Ait-Sahalia and Lo (2000) propose a nonparametric VaR measure that incorporates economic valuation through the state-price density associated with the underlying price processes. Ait-Sahalia and Lo (1998) propose to estimate an option-pricing formula $\hat{H}(\cdot)$ which has the structure of the Black–Scholes formula but where the volatility is estimated nonparametrically, that is $H_{BS}(F_{t,\tau}, X, \tau, r_{t,\tau}; \sigma(X/F_{t,\tau}, \tau))$, where $F_{t,\tau}$ is the forward price of the asset (S&P 500 index) at time t for time-to-maturity τ , X is the exercise price, r denotes the interest rate, and $\sigma(\cdot)$ is the volatility function. The estimated option-price function can be differentiated twice with respect to the strike price X to obtain $\partial^2 \hat{H}(\cdot)/\partial X^2$, and then the state-price density by multiplying this second derivative by $\exp(r_{t,\tau}\tau)$. Ait-Sahalia and Lo (2000) used this estimated state-price density to compute an economic VaR but we can of course use it to compute the excess ES.

The main point of Ait-Sahalia and Lo (2000) was to arrive at a VaR value that is adjusted for risk and time preferences and for changes in economic conditions. We have the same objective but we differ from their work in one important aspect. We estimate the risk-neutral density from a panel of equity returns and distort the objective probabilities through a minimum-discrepancy procedure. We apply the two methodologies for the period January 1996 to April 2014. Figure 4 features both tail-risk measures. Despite several methodological and empirical differences between the two measures, Figure 4 reveals that both time series tend to move together with many coincident peaks and troughs.

To analyze more precisely the similarities between the two measures we conduct an analysis of implied market premium. We first measure the hedging capacity (or insurance value) of all individual stocks with code 10-11 in CRSP over the period (i.e., their contemporaneous betas with respect to the tail-risk measure, $r_{i,t} = \alpha_i + \beta_i \text{TR}_{i,t}$ ¹⁷) and then sort

¹⁷ This is to be distinguished from the predictive betas computed in Kelly and Jiang (2014), $r_{i,t+1} = \alpha_i + \beta_i \text{TR}_{i,t}$, which measure exposure to tail risk.

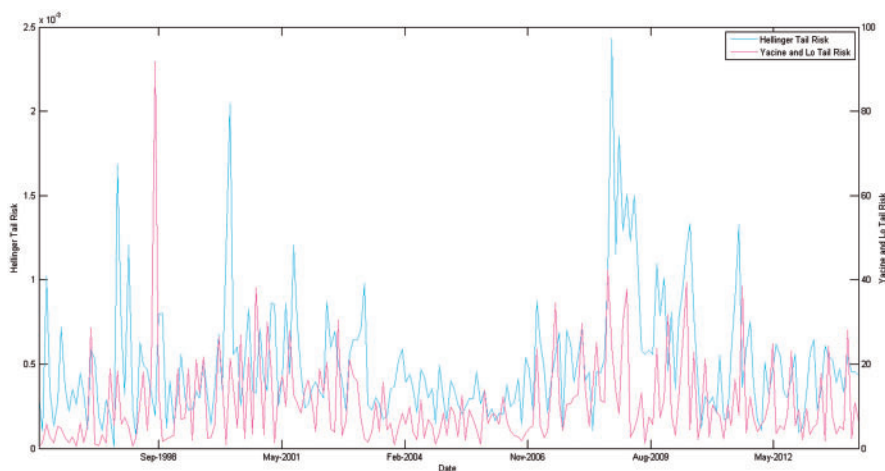


Figure 4. This figure plots two series of tail risk. The blue line corresponds to the tail risk computed from the five principal components extracted monthly from the 25 Fama–French size and book-to-market portfolios with the estimated Hellinger risk-neutral density ($\gamma = -0.5$). The red line features the corresponding tail-risk measure based on a state-price density obtained with the Yacine and Lo (1998, 2000) methodology applied to a panel of S&P 500 index options. The sample period ranges from January 1996 to April 2014.

these betas into 10 portfolios for the lowest hedging portfolio to the highest one. We then compute the post-formation returns associated with these portfolios over the next month. The results are reported in Table 3. There are two panels. The upper panel reports the option-implied tail risk while the lower one features our tail-risk measure with the Hellinger discrepancy. For each panel, we report several measures of returns. In the first line, we present the raw average returns for the 10 portfolios and for the difference between the highest hedging and the lowest hedging portfolios (last column High-Low). For the other lines we report the returns after controlling for other factors, namely the three Fama–French factors, to which we add momentum, liquidity, and volatility one by one.

First, looking at the first line which reports the raw average returns, it is remarkable to see that the magnitudes of the High-Low column are very similar and share the same signs and statistical significance. The difference is -3.73 for the option-based measure while it is -3.06 for the Hellinger measure. This suggests that the crash perceptions embedded in the option prices are stronger than in our nonparametric distortion. After controlling for the other factors, all the signs remain negative except when we add volatility. In terms of statistical significance the option-implied high minus low returns are estimated more precisely than the nonparametric measure, which makes sense since we do not incorporate market prices of options.

There is evidently some short-run correlation between volatility and tail risk. One can think of an equilibrium explanation in a consumption-based asset pricing model as in Farago and Tedongap (2015). They propose an intertemporal equilibrium asset pricing model featuring disappointment aversion and changing macroeconomic uncertainty (consumption volatility) where besides the market return and market volatility, three disappointment-related factors are also priced: a disappointment factor, a market downside

Table 3. Option vs. Hellinger sorted portfolios

Portfolio	Option implied tail risk										
	Low	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	High	High-Low
Average return	4.63 (2.57)	2.13 (2.29)	2.48 (2.49)	1.26 (1.54)	1.78 (1.86)	1.17 (1.55)	0.85 (1.33)	0.97 (1.64)	0.81 (1.23)	0.90 (1.72)	-3.73 (-2.35)
FF3	3.51 (2.22)	1.47 (1.53)	1.74 (1.76)	0.50 (0.62)	1.15 (1.14)	0.55 (0.72)	0.31 (0.44)	0.40 (0.71)	0.36 (0.52)	0.56 (1.00)	-2.96 (-2.15)
FF3+MOM	3.72 (2.02)	1.32 (1.41)	1.54 (1.59)	0.47 (0.60)	0.93 (1.01)	0.48 (0.65)	0.30 (0.45)	0.45 (0.81)	0.31 (0.48)	0.64 (1.19)	-3.08 (-1.84)
FF3+MOM+LIQ	4.43 (1.72)	1.37 (1.26)	1.59 (1.42)	0.41 (0.46)	1.08 (0.91)	0.60 (0.68)	0.37 (0.49)	0.41 (0.67)	0.50 (0.62)	0.63 (1.03)	-3.80 (-1.60)
FF3+MOM+LIQ+VOL	-0.89 (-0.33)	0.92 (0.51)	0.85 (0.49)	-0.19 (-0.12)	-0.37 (-0.25)	-0.22 (-0.16)	0.71 (0.59)	0.13 (0.12)	0.03 (0.03)	1.43 (1.56)	2.32 (1.02)
Portfolio	Hellinger tail risk										
	Low	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	High	High-Low
Average return	4.48 (2.52)	2.71 (2.44)	2.20 (2.27)	1.22 (1.53)	1.88 (1.96)	0.84 (1.25)	0.73 (1.20)	0.63 (1.12)	0.88 (1.52)	1.42 (2.18)	-3.06 (-2.03)
FF3	3.37 (2.17)	1.92 (1.72)	1.48 (1.52)	0.49 (0.62)	1.27 (1.29)	0.30 (0.43)	0.22 (0.36)	0.21 (0.36)	0.32 (0.54)	0.97 (1.35)	-2.40 (-1.87)
FF3+MOM	3.67 (2.04)	1.53 (1.44)	1.28 (1.36)	0.37 (0.48)	1.15 (1.28)	0.26 (0.38)	0.19 (0.33)	0.17 (0.30)	0.48 (0.82)	1.05 (1.51)	-2.63 (-1.67)
FF3+MOM+LIQ	4.39 (1.73)	1.74 (1.34)	1.34 (1.21)	0.36 (0.42)	1.35 (1.17)	0.19 (0.25)	0.15 (0.23)	0.19 (0.29)	0.44 (0.66)	1.25 (1.58)	-3.14 (-1.36)
FF3+MOM+LIQ+VOL	-0.01 (-0.00)	0.26 (0.15)	0.90 (0.51)	-0.09 (-0.07)	0.13 (0.09)	0.02 (0.01)	-0.41 (-0.37)	-0.02 (-0.02)	0.15 (0.14)	1.48 (1.16)	1.49 (0.72)

Notes: The table reports the returns of decile portfolios of all CRSP stocks with code 10-11 sorted according to their tail-risk hedging capacity. In Panel A, the tail-risk factor is computed according to the nonparametric methodology of Ait-Sahalia and Lo (2000) and uses option prices. In Panel B, the tail-risk factor is computed from the returns of the first five principal components extracted from the 25 size and book-to-market portfolios according to the minimum discrepancy procedure to risk-adjust returns. For each month in our sample from January 1996 to April 2014, we sort the stocks and track their returns one month post-formation. In the first line we report the average portfolio returns. In the following lines we report the t 's of regressions where we control for the factors indicated in the first column. Newey-West t -statistics computed with one lag are reported between parentheses.

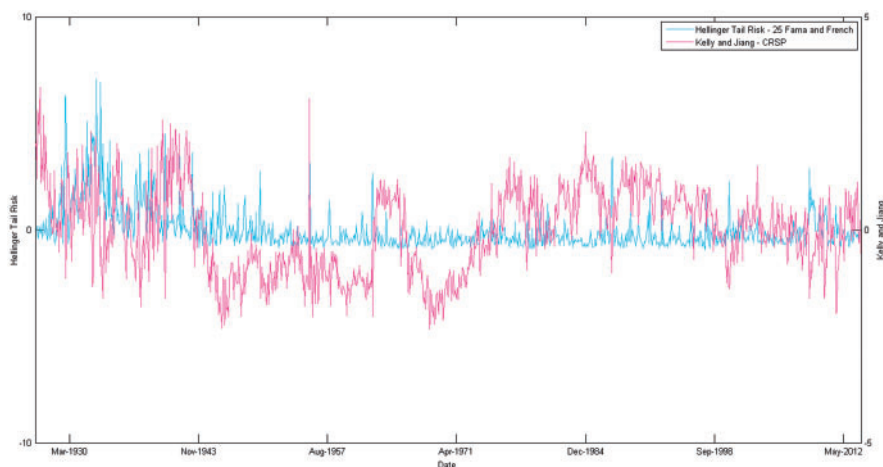


Figure 5. This figure plots two series of tail risk. The blue line corresponds to the tail risk computed from the five principal components extracted monthly from the 25 Fama and French size and book-to-market portfolios with the estimated Hellingger risk-neutral density ($\gamma = -0.5$). The red line features the tail-risk measure proposed by Kelly and Jiang (2013) based on a Hill estimator computed from the whole cross-section of CRSP securities available at each time period. The sample period ranges from January 1996 to December 2012.

factor, and a volatility downside factor. The last factor represents changes in market volatility in disappointing states and calls for a particular premium. Our linear analysis cannot disentangle these downside effects, hence the correlation between the volatility factor and the tail-risk factor.

We believe that these results show that our nonparametric tail-risk measure goes a long way in capturing the risk adjustment ingrained in option prices. Therefore, it can supplement the option-based tail-risk measures for markets where liquid option markets are not present and for periods where these derivative securities did not exist but where tail risk was prominent, such as the Great Depression of 1929–1939. Since our measure is based on a panel of equity returns we need to understand its relation with the Kelly and Jiang (2014) tail-risk measure, especially since they appear to be uncorrelated.

2.3. A Detailed Comparison with the Kelly and Jiang (2014) Tail-Risk Measure

Figure 5 plots the tail-risk measure based on our expected excess shortfall methodology together with the Kelly and Jiang (2014) measure. The two series are quite distinct and seem to relate negatively when big shocks hit the market. Kelly and Jiang (2014) assume that the lower tail of any asset return i behaves according to the following law:

$$P(R_{i,t+1} < r | R_{i,t+1} < u_t \text{ and } \mathcal{F}_t) = \left(\frac{r}{u_t}\right)^{-a_t/\lambda_t}, \quad (17)$$

with $r < u_t < 0$. The authors estimate the time-varying tail exponent by the Hill estimator:

$$\lambda_t^{\text{Hill}} = \frac{1}{K_t} \sum_{k=1}^{K_t} \ln \frac{R_{k,t}}{u_t}, \quad (18)$$

where $R_{k,t}$ is the k th daily return that falls below an extreme value threshold u_t during month t , and K_t is the total number of such exceedances in the cross-section within month t . They find that the tail exponent is highly persistent and features a high degree of commonality across firms.

Although the KJ estimator is based on an assumption about the statistical behavior of the tail of the cross-section distribution of returns, our estimator relies on a risk adjustment that distorts the returns of the chosen basis assets according to a set of Euler conditions. This adds a lot of variability to our tail-risk estimator.

Although our tail-risk factor and the Kelly and Jiang (2014) tail-risk measure are computed from a cross-section of equity returns they differ in several aspects. An important difference relates to the information used by each method. Each month, Kelly and Jiang (2014) compute their tail-risk measure based on the whole set of individual raw returns below a threshold of 5% in the elapsed month. To set ideas if we have 5000 individual securities, the procedure uses 5% of 100,000 observations (5000×20 days), that is 5000 observations. Our measure starts from a set of 25 portfolios sorted by size and book-to-market value, collects their returns over the last 30 days (a month and a half) and then extracts five principal components of these 25 series of 30 days, leaving us with five series of 30 observations. We then compute our excess ES with the risk-adjusted returns at a level of 10% for each principal component and take the average of the five shortfall measures. Therefore, there is a fundamental difference since the Kelly and Jiang (2014) method extracts a common component among the say 5000 tail observations while our procedure selects extreme observations after a double reduction in information by first looking at portfolios and then taking principal components of these portfolios. Is this contraction of information the main reason for the fundamental difference between the two measures of tail risk?

To answer this question we start by running our procedure with all individual securities. We compute the five principal components out of the whole universe of available stocks every month for a window of 30 days. The robustness of the procedure is evaluated as in Section 2.2 with the returns of 10 portfolios sorted according to the tail-risk hedging capacity and the resulting high minus low returns. To benchmark the results obtained with all securities, we first compute the portfolio returns over the 1926–2014 period with our original data set based on the 25 size and book-to-market portfolios. In Table 4, we report in the last column the high minus low returns earned post-formation for a one-month holding period (Panel A) and for a one-year holding period (Panel B). For the short-period returns, all differences are negative, of similar magnitude across the various controls for additional factors and statistically significant except for the last line where we add a volatility factor. The results are similar for the yearly returns but the difference is now positive and insignificant when we add the volatility factor. Table 5 contains the results of the same analysis when the five principal components are computed from the whole universe of equities available in CRSP. Results for both short and long holding periods are very similar. Moreover, we have conducted the same analysis directly on the 25 size and book-to-market portfolios instead of their five principal components and results were also very similar. Therefore, we can conclude confidently that our methodology is robust to the amount of cross-sectional information used to build the tail-risk measure.

Another source of divergence could be due to our risk-neutralization. To verify if it is, we define a standardized tail-risk measure based on the average of the excess ES in

Table 4. Hellinger sorted portfolios—25 size and book-to-market portfolios

Portfolio	Panel A: One-month holding period										
	Low	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	High	High-Low
Average return	2.34 (4.18)	1.38 (3.57)	1.04 (3.13)	0.74 (2.51)	0.88 (2.83)	0.56 (2.16)	0.48 (1.99)	0.38 (1.74)	0.53 (2.39)	0.82 (3.30)	-1.52 (-3.41)
FF3	1.21 (3.20)	0.31 (1.85)	0.05 (0.39)	-0.21 (-2.13)	-0.08 (-0.66)	-0.28 (-2.90)	-0.31 (-3.35)	-0.35 (-4.34)	-0.18 (-1.57)	0.06 (0.46)	-1.15 (-3.09)
FF3+MOM	1.24 (3.17)	0.32 (1.96)	0.07 (0.51)	-0.19 (-1.96)	-0.05 (-0.41)	-0.27 (-2.75)	-0.30 (-3.22)	-0.33 (-4.05)	-0.17 (-1.53)	0.05 (0.34)	-1.20 (-3.10)
FF3+MOM+LIQ	1.35 (2.90)	0.32 (2.03)	0.07 (0.54)	-0.18 (-1.87)	-0.05 (-0.41)	-0.27 (-2.72)	-0.28 (-3.04)	-0.32 (-3.88)	-0.16 (-1.36)	0.06 (0.48)	-1.28 (-2.79)
FF3+MOM+LIQ+VOL	0.87 (2.02)	-0.02 (-0.13)	-0.05 (-0.32)	-0.31 (-2.62)	-0.14 (-1.03)	-0.21 (-1.98)	-0.34 (-3.00)	-0.31 (-3.20)	-0.08 (-0.53)	0.04 (0.25)	-0.83 (-1.80)

Portfolio	Panel B: One year holding period										
	Low	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	High	High-Low
Average return	20.83 (3.49)	11.19 (3.04)	9.33 (2.82)	6.09 (2.29)	6.06 (2.43)	4.55 (1.87)	3.57 (1.55)	3.04 (1.44)	4.60 (2.04)	6.99 (2.57)	-13.84 (-3.10)
FF3	0.77 (0.25)	-3.51 (-2.18)	-4.00 (-2.61)	-5.62 (-4.64)	-4.91 (-4.09)	-5.90 (-4.32)	-6.61 (-5.05)	-6.06 (-5.14)	-4.82 (-3.79)	-2.40 (-1.70)	-3.17 (-1.16)
FF3+MOM	2.60 (0.93)	-2.32 (-1.53)	-3.34 (-2.26)	-5.15 (-4.68)	-4.89 (-4.42)	-5.40 (-4.27)	-6.10 (-4.99)	-5.34 (-4.87)	-4.11 (-3.23)	-2.39 (-1.64)	-4.99 (-2.01)
FF3+MOM+LIQ	4.04 (1.49)	-2.26 (-1.81)	-3.39 (-2.36)	-5.23 (-5.34)	-5.04 (-4.60)	-5.25 (-4.04)	-5.94 (-4.58)	-5.08 (-4.35)	-3.52 (-2.56)	-0.94 (-0.53)	-4.98 (-2.05)
FF3+MOM+LIQ+VOL	-15.34 (-1.38)	-10.25 (-1.89)	-10.61 (-2.64)	-10.54 (-3.26)	-11.14 (-4.60)	-9.02 (-3.82)	-8.52 (-3.81)	-6.77 (-3.58)	-8.35 (-4.41)	-3.79 (-1.33)	11.56 (1.03)

Notes: Returns are reported for decile portfolios of CRSP stocks with code 10-11 sorted according to their tail-risk hedging capacity. The tail-risk factor is computed from the returns of the first five principal components extracted from the 25 size and book-to-market portfolios. The returns are adjusted according to the minimum discrepancy procedure with a gamma of -0.5 (Hellinger measure). For each month in our sample from February 1967 to December 2013, we sort the stocks and track their returns one-month post formation (Panel A) or one-year post formation (Panel B). In the first line we report the average portfolio returns. In the following lines we report the t 's of regressions where we control for the factors indicated in the first column. Newey-West t -statistics reported between parentheses are computed with one lag for monthly results and 12 lags for yearly results.

Table 5. Hellinger sorted portfolios—all securities

Portfolio	Panel A: One month holding period										
	Low	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	High	High-Low
Average return	2.24 (4.09)	1.12 (3.21)	1.02 (3.31)	0.93 (3.08)	0.62 (2.35)	0.66 (2.39)	0.54 (2.25)	0.60 (2.21)	0.45 (2.09)	0.97 (3.38)	-1.28 (-2.98)
FF3	1.13 (3.02)	0.08 (0.62)	0.04 (0.36)	0.02 (0.14)	-0.24 (-2.31)	-0.20 (-1.97)	-0.25 (-2.85)	-0.24 (-2.28)	-0.25 (-2.83)	0.14 (0.92)	-0.98 (-2.59)
FF3+MOM	1.16 (3.01)	0.11 (0.78)	0.07 (0.61)	0.03 (0.26)	-0.23 (-2.16)	-0.18 (-1.73)	-0.22 (-2.58)	-0.22 (-1.99)	-0.26 (-2.96)	0.11 (0.73)	-1.05 (-2.69)
FF3+MOM+LIQ	1.25 (2.72)	0.11 (0.81)	0.08 (0.69)	0.04 (0.33)	-0.23 (-2.30)	-0.18 (-1.71)	-0.22 (-2.57)	-0.19 (-1.58)	-0.25 (-2.84)	0.12 (0.77)	-1.13 (-2.42)
FF3+MOM+LIQ+VOL	0.90 (2.21)	-0.04 (-0.25)	-0.13 (-0.86)	-0.17 (-1.09)	-0.32 (-2.73)	-0.26 (-2.58)	-0.21 (-2.01)	-0.16 (-1.32)	-0.29 (-2.78)	0.13 (0.65)	-0.77 (-1.73)

Portfolio	Panel B: One year holding period										
	Low	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	High	High-Low
Average return	18.93 (3.70)	10.33 (2.98)	8.18 (2.73)	6.26 (2.28)	5.64 (2.05)	4.64 (1.85)	4.56 (1.90)	4.38 (1.85)	4.26 (1.94)	9.05 (2.87)	-9.88 (-2.92)
FF3	1.90 (0.64)	-4.33 (-2.88)	-4.70 (-3.52)	-5.82 (-4.68)	-5.68 (-3.46)	-5.80 (-4.17)	-5.63 (-4.11)	-5.82 (-4.51)	-4.86 (-3.54)	-2.30 (-1.33)	-4.20 (-1.31)
FF3+MOM	1.54 (0.67)	-3.26 (-2.41)	-3.94 (-3.22)	-5.34 (-4.59)	-4.46 (-2.86)	-5.13 (-3.69)	-5.04 (-3.70)	-5.17 (-4.13)	-3.99 (-3.00)	-1.67 (-0.90)	-3.21 (-1.32)
FF3+MOM+LIQ	2.91 (1.18)	-3.13 (-2.49)	-3.61 (-2.69)	-4.68 (-3.92)	-4.71 (-2.89)	-5.49 (-4.12)	-5.15 (-3.73)	-4.89 (-3.95)	-3.66 (-2.81)	-0.20 (-0.10)	-3.10 (-1.16)
FF3+MOM+LIQ+VOL	-11.34 (-1.61)	-10.15 (-2.41)	-9.56 (-3.04)	-10.18 (-3.43)	-11.58 (-3.42)	-8.21 (-3.33)	-8.04 (-3.36)	-9.81 (-4.23)	-9.95 (-4.77)	-5.50 (-1.30)	5.85 (1.08)

Notes: Returns are reported for decile portfolios of CRSP stocks with code 10-11 sorted according to their tail-risk hedging capacity. The tail-risk factor is computed from the returns of the first five principal components extracted from the whole set of securities available at any point in time. The returns are adjusted according to the minimum discrepancy procedure with a gamma of -0.5 (Hellinger measure). For each month in our sample from February 1967 to December 2013, we sort the stocks and track their returns one-month post formation (Panel A) or one-year post formation (Panel B). In the first line we report the average portfolio returns. In the following lines we report the t 's of regressions where we control for the factors indicated in the first column. Newey-West t -statistics reported between parentheses are computed with one lag for monthly results and 12 lags for yearly results.

Equation (1) for the raw returns of the five principal components, a so-called objective counterpart to our risk-neutral measure. We proceed similarly to sort the cross-section of returns in 10 portfolios according to their standardized tail-risk hedging sensitivity and compute the post-formation returns of the portfolios as before. Table 6 presents the results with this alternative objective tail-risk measure. We note that both in terms of magnitude and statistical significance results are very different from our reported figures in Table 4, providing some evidence of the important contribution of the risk-neutralization procedure.

We conclude this section by some interesting observations about the Kelly and Jiang (2014) tail-risk measure. If we express the law of tail returns in terms of gross returns instead of net returns, that is $1 + R_{k,t}$, $1 + r$ and $1 + u$, then the Hill estimator will be approximately, for small daily returns:

$$\lambda_t^{\text{Hill}} = \frac{1}{K_t} \sum_{k=1}^{K_t} [R_{k,t} - u_t], \quad (19)$$

Based on this new Hill estimator, we compute the tail-risk equivalent KJ series with the whole cross-section at any point in time. We plot in Figure 6 this series together with our Hellinger measure of tail risk based on the five principal components of the whole cross-section of returns. Despite the different methodologies and the risk-neutralization, the two series behave much more similarly than before. Averaging the difference of the logarithms of the returns or the difference of the returns makes a huge difference on the property of the tail-risk series. Of course this new Hill estimator is closer to our excess ES measure, the main difference being the fact that our measure is based on risk-adjusted returns of more aggregated information. The graphs suggest that these two elements, risk neutralization, and aggregation of information, are important for the magnitude of the fluctuations in both series, sometimes enhancing them sometimes dampening them. There is not a clear consistent pattern over time.

3 Predictive Properties of Tail Risk

Predictability of market returns or other financial and macroeconomic variables is another way to assess the usefulness of our tail-risk measure. A key recent paper by Welch and Goyal (2008) studies comprehensively the predictive power of variables that have been suggested by the academic literature to be good predictors of the equity premium. Their conclusions are rather negative both in-sample and out-of-sample. They find that models have a poor and unstable predictive performance. Therefore, we want to submit our measure to the same type of analysis and determine whether it adds to the main predictors established in the literature. We explore in turn market returns, macroeconomic responses to tail-risk shocks, and finally how tail-risk anticipates business cycle fluctuations.

3.1. Stock Markets Returns

We provide two sets of predictive regressions. First, in Table 7, we consider simple regressions where we control for market returns (12 lags): $r_{[t,t+k]} = \alpha + \beta \text{TR}_t + \gamma r_{[t-k,t]} + u_t$, where TR_t denotes the tail-risk measure. Our goal is to compare the predictive power of our measure relative to the other proposed tail-risk measures. This has two advantages. It is

Table 6. Tail-risk measure under objective excess shortfall—sorted portfolios

Portfolio	Panel A: One-month holding period										
	Low	2	3	4	5	6	7	8	9	High	High-Low
Average return	1.84 (4.26)	1.37 (2.91)	1.11 (3.25)	0.77 (2.59)	0.59 (2.28)	0.61 (2.46)	0.65 (2.67)	0.49 (2.05)	0.54 (2.26)	1.17 (3.85)	-0.67 (-2.50)
FF3	0.74 (3.53)	0.36 (1.06)	0.12 (0.80)	-0.16 (-1.42)	-0.25 (-2.68)	-0.18 (-1.73)	-0.13 (-1.35)	-0.30 (-3.71)	-0.25 (-2.58)	0.28 (1.79)	-0.46 (-1.95)
FF3+MOM	0.74 (3.61)	0.38 (1.09)	0.13 (0.85)	-0.15 (-1.32)	-0.24 (-2.46)	-0.17 (-1.53)	-0.10 (-1.06)	-0.28 (-3.46)	-0.23 (-2.40)	0.29 (1.83)	-0.46 (-1.99)
FF3+MOM+LIQ	0.75 (3.65)	0.46 (1.08)	0.14 (0.92)	-0.14 (-1.21)	-0.22 (-2.30)	-0.15 (-1.27)	-0.12 (-1.36)	-0.27 (-3.36)	-0.23 (-2.36)	0.33 (2.02)	-0.42 (-1.81)
FF3+MOM+LIQ+VOL	0.30 (1.30)	0.06 (0.18)	0.04 (0.28)	-0.25 (-2.15)	-0.26 (-2.13)	-0.21 (-1.33)	-0.16 (-1.30)	-0.34 (-3.63)	-0.17 (-1.31)	0.43 (2.17)	0.12 (0.46)

Portfolio	Panel B: One year holding period										
	Low	2	3	4	5	6	7	8	9	High	High-Low
Average return	17.97 (3.47)	10.17 (3.00)	8.49 (2.83)	5.99 (2.18)	4.56 (1.88)	4.89 (1.93)	4.10 (1.84)	4.11 (1.89)	5.46 (2.24)	10.50 (3.20)	-7.47 (-2.38)
FF3	-0.51 (-0.18)	-3.39 (-2.02)	-4.01 (-2.49)	-5.87 (-4.46)	-5.98 (-4.45)	-5.52 (-4.04)	-5.68 (-5.07)	-5.61 (-5.60)	-4.77 (-4.37)	-1.70 (-1.05)	-1.19 (-0.45)
FF3+MOM	-0.10 (-0.04)	-2.21 (-1.40)	-3.35 (-2.24)	-5.39 (-4.28)	-5.45 (-4.08)	-5.09 (-4.16)	-5.03 (-4.87)	-5.07 (-5.51)	-4.48 (-4.18)	-0.28 (-0.18)	-0.17 (-0.09)
FF3+MOM+LIQ	0.62 (0.27)	-1.69 (-1.02)	-3.00 (-1.88)	-5.35 (-4.16)	-5.67 (-4.11)	-4.70 (-3.43)	-5.30 (-5.56)	-5.07 (-5.48)	-3.84 (-3.49)	1.41 (0.85)	0.79 (0.42)
FF3+MOM+LIQ+VOL	-18.74 (-1.93)	-13.20 (-2.19)	-10.47 (-2.55)	-11.27 (-3.56)	-8.82 (-3.95)	-7.98 (-2.87)	-7.41 (-4.02)	-8.03 (-4.66)	-6.27 (-3.59)	-2.15 (-0.88)	16.59 (1.75)

Notes: Returns are reported for decile portfolios of CRSP stocks with code 10-11 sorted according to their tail-risk hedging capacity with respect to the ratio between the risk-neutral and objective tail-risk measures. The tail-risk factor is computed from the returns of the first five principal components extracted from the 25 size and book-to-market portfolios. The objective tail-risk ES is calculated assuming a homogeneous probability density function. For each month in our sample from February 1967 to December 2013, we sort the stocks and track their returns one-month post formation (Panel A) or one-year post formation (Panel B). In the first line we report the average portfolio returns. In the following lines we report the α 's of regressions where we control for the factors indicated in the first column. Newey-West t -statistics reported between parentheses are computed with one lag for monthly results and 12 lags for yearly results.

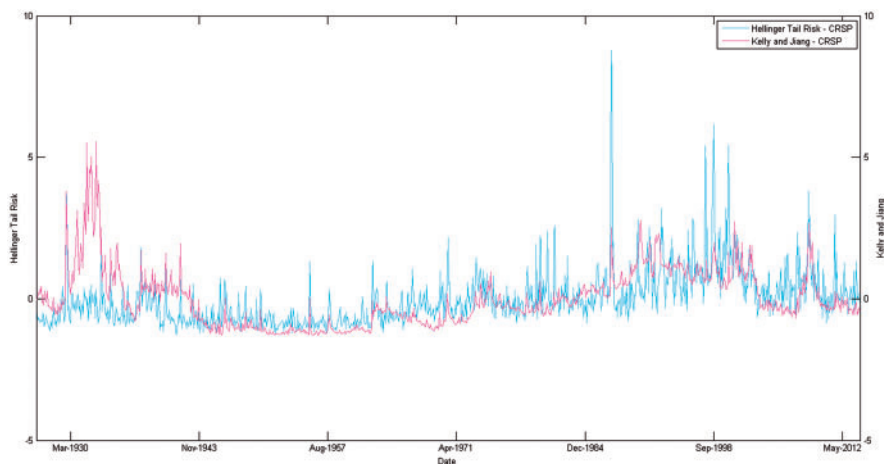


Figure 6. This figure plots two series of tail risk. The blue line corresponds to the tail risk computed from the five principal components extracted monthly from the whole cross-section of CRSP securities available at each time period with the estimated Hellinger risk-neutral density ($\gamma = -0.5$). The red line features the tail-risk measure proposed by Kelly and Jiang (2013) based on the Hill estimator computed from the whole cross-section of CRSP securities available at each time period, based on the Hill formula featured in Equation (19). The sample period ranges from January 1996 to December 2012.

a test with respect to the direct competition but it is also a robustness test with respect to sub-samples of the long sample 1926–2014 since the various tail-risk measures are available over different periods. We choose the CRSP value-weighted market returns index as our target.¹⁸ Overall, we note that the estimated regression coefficients are positive, meaning that increases in current tail risk are associated with future higher market returns. Also, for most of the samples over which we perform the regressions the estimated coefficients are statistically significant for an interval of 2–6 months. The most robust performance comes from the long horizons starting in the 1960s. This is not the case of the other tail-risk measures. The CATFIN index does not show any forecasting power while the Bloom volatility index is forecasting at a very short horizon of 1 or 2 months. The VIX has some predictive ability in the medium-term, while the Kelly–Jiang tail-risk measure starts showing some predictive power after 6 months. The best predictor appears to be the BTX measure from Bollerslev, Todorov, and Xu (2015). From the third month on, we see a statistically significant relation between future CRSP returns and the option-based tail-risk measure. We also observe a growing pattern in the coefficients. This is also apparent in our Hellinger tail-risk measure, which could be due to the period starting in 1996, because this pattern is also present for the regressions with the VIX and not for the other horizons.

We continue our predictive exercise in Table 8, by adding many of the predictive candidates used in Welch and Goyal (2008) to the previous regression with our Hellinger tail-risk measure and lagged returns for the long sample (1962–2012), that is $r_{[t,t+k]} = \alpha + \beta \text{TR}_t + \gamma r_{[t-k,t]} + \delta \text{GW}_t + u_t$. In Panel A, of the seven predictors considered (book-to-market, dividend payout, earnings price ratio, dividend price ratio, dividend yield,

18 Since results with the S&P500 were very similar, we omitted them for space considerations.

Table 7. Predicting market returns

Panel A: Hellinger tail risk										
Forecast	TR	<i>t</i>	TR	<i>t</i>	TR	<i>t</i>	TR	<i>t</i>	TR	<i>t</i>
1	0.10	(2.90)	0.10	(1.36)	0.10	(2.43)	0.03	(0.52)	0.08	(1.81)
2	0.10	(2.28)	0.11	(1.35)	0.12	(2.45)	0.08	(1.06)	0.10	(1.77)
3	0.10	(2.51)	0.17	(1.97)	0.15	(3.09)	0.12	(1.53)	0.13	(2.35)
4	0.08	(1.90)	0.21	(2.21)	0.15	(2.93)	0.15	(1.78)	0.12	(2.06)
5	0.06	(1.50)	0.23	(2.20)	0.14	(2.62)	0.17	(1.94)	0.11	(1.80)
6	0.03	(0.59)	0.21	(1.69)	0.12	(1.97)	0.15	(1.44)	0.08	(1.21)
7	0.03	(0.57)	0.22	(1.71)	0.12	(1.97)	0.17	(1.52)	0.09	(1.27)
8	0.01	(0.21)	0.21	(1.47)	0.11	(1.64)	0.14	(1.15)	0.08	(1.05)
9	-0.02	(-0.33)	0.18	(1.14)	0.09	(1.20)	0.11	(0.81)	0.06	(0.67)
10	-0.03	(-0.49)	0.20	(1.21)	0.09	(1.08)	0.12	(0.87)	0.06	(0.63)
11	-0.03	(-0.48)	0.20	(1.24)	0.09	(1.03)	0.12	(0.87)	0.05	(0.58)
12	-0.02	(-0.42)	0.19	(1.24)	0.08	(1.00)	0.11	(0.81)	0.05	(0.55)

Panel B: Other tail-risk measures										
Forecast	Bloom	<i>t</i>	BTX	<i>t</i>	KJ	<i>t</i>	VIX	<i>t</i>	CATFIN	<i>t</i>
1	0.08	(1.98)	0.11	(1.13)	0.05	(1.28)	0.07	(1.00)	0.04	(0.73)
2	0.11	(2.11)	0.15	(1.21)	0.09	(1.67)	0.11	(1.15)	0.04	(0.58)
3	0.10	(1.74)	0.19	(1.95)	0.10	(1.65)	0.15	(1.63)	0.09	(1.08)
4	0.09	(1.37)	0.26	(3.33)	0.12	(1.77)	0.17	(2.01)	0.07	(0.84)
5	0.08	(1.15)	0.30	(3.61)	0.15	(1.89)	0.21	(2.45)	0.08	(0.98)
6	0.08	(0.96)	0.29	(2.87)	0.17	(2.04)	0.22	(2.52)	0.06	(0.75)
7	0.07	(0.77)	0.31	(2.48)	0.19	(2.13)	0.23	(2.31)	0.06	(0.90)
8	0.04	(0.41)	0.30	(2.10)	0.21	(2.22)	0.22	(2.04)	0.06	(0.85)
9	0.03	(0.32)	0.30	(2.11)	0.22	(2.27)	0.20	(1.78)	0.05	(0.65)
10	0.02	(0.20)	0.30	(2.05)	0.23	(2.32)	0.20	(1.67)	0.06	(0.79)
11	0.01	(0.08)	0.32	(2.19)	0.25	(2.49)	0.20	(1.58)	0.06	(0.76)
12	0.01	(0.05)	0.34	(2.29)	0.27	(2.69)	0.21	(1.53)	0.07	(0.88)

Notes: Returns of the CRSP value-weighted market index returns are predicted with the following regressions: $r_{t,t+k} = \alpha + \beta TR_{t-1} + \gamma r_{t-k,t} + u_t$. We consider the following financial “tail-risk” measures (sample in parenthesis) as explanatory variables: Hellinger Tail Risk; Bloom (2009) volatility (07/1962–06/2008); Kelly and Jiang (2014) tail-risk measure (01/1963–12/2010); Allen, Bale, and Tang (2012) systemic risk measure (01/1973–12/2012); the VIX index (01/1990–04/2014); and Bollerslev, Todorov, and Xu (2015) tail-risk measure (01/1996–08/2013). In Panel A, we run the same regression for the Hellinger tail-risk measure for various samples corresponding to the respective measures considered in Panel B. *t*-Statistics are calculated using Newey–West variance matrix with 2–24 lags for 1–12 forecasting horizon.

stock variance, default yield spread), only the dividend price ratio and the dividend yield appear to be significantly related to future stock market returns for all horizons, as it has been often established. However, it does not take away the predictive power of tail risk for the 3- to 8-month horizons. The results for the default yield spread are particularly interesting. It shows some significant predictive ability for the 6- to 12-month horizons, but it takes away the predictive significant relation between tail risk and future market returns. This

Table 8. Welsh and Goyal (2008) regressions

Panel A: Hellinger tail risk

Forecast	TR (BM)		TR (DP)		TR (EPR)		TR (DPR)		TR (DY)		TR (SV)		TR (DYS)	
	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>
1	0.03	(0.58)	0.02	(0.42)	0.04	(0.77)	0.03	(0.59)	0.03	(0.59)	0.12	(0.59)	0.12	(2.19)
2	0.09	(1.62)	0.08	(1.36)	0.11	(1.96)	0.09	(1.61)	0.09	(1.63)	0.16	(1.63)	0.16	(3.21)
3	0.14	(2.62)	0.11	(2.17)	0.15	(2.96)	0.14	(2.65)	0.14	(2.70)	0.19	(2.70)	0.19	(3.19)
4	0.14	(2.59)	0.11	(2.01)	0.15	(2.86)	0.14	(2.60)	0.14	(2.68)	0.17	(2.68)	0.17	(2.82)
5	0.12	(2.55)	0.09	(1.82)	0.13	(2.82)	0.12	(2.59)	0.13	(2.73)	0.14	(2.73)	0.14	(2.31)
6	0.11	(2.42)	0.08	(1.61)	0.12	(2.65)	0.11	(2.50)	0.12	(2.65)	0.12	(2.65)	0.12	(2.00)
7	0.10	(2.01)	0.07	(1.38)	0.11	(2.25)	0.10	(2.13)	0.11	(2.28)	0.09	(2.28)	0.09	(1.44)
8	0.10	(1.98)	0.04	(1.32)	0.11	(2.21)	0.10	(2.10)	0.11	(2.25)	0.08	(2.25)	0.08	(1.31)
9	0.10	(1.74)	0.06	(1.20)	0.11	(2.00)	0.10	(1.86)	0.11	(2.00)	0.07	(2.00)	0.07	(1.11)
10	0.08	(1.35)	0.05	(0.82)	0.10	(1.60)	0.09	(1.46)	0.09	(1.60)	0.05	(1.60)	0.05	(0.78)
11	0.08	(1.18)	0.04	(0.66)	0.09	(1.42)	0.08	(1.28)	0.09	(1.42)	0.04	(1.42)	0.04	(0.58)
12	0.08	(1.13)	0.04	(0.58)	0.09	(1.39)	0.08	(1.22)	0.09	(1.39)	0.03	(1.39)	0.03	(0.48)

Forecast	BM		DP		EPR		DPR		DY		SV		DYS	
	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>	<i>t</i>
1	0.06	(1.30)	0.02	(0.35)	0.07	(1.39)	0.09	(2.16)	0.09	(2.17)	-0.18	(2.17)	-0.18	(-3.79)
2	0.08	(1.45)	0.03	(0.45)	0.10	(1.40)	0.13	(2.40)	0.13	(2.41)	-0.15	(2.41)	-0.15	(-2.22)
3	0.11	(1.61)	0.04	(0.69)	0.12	(1.44)	0.16	(2.59)	0.16	(2.58)	-0.13	(2.58)	-0.13	(-1.45)
4	0.13	(1.59)	0.06	(0.92)	0.13	(1.32)	0.19	(2.54)	0.19	(2.55)	-0.10	(2.55)	-0.10	(-0.95)
5	0.14	(1.56)	0.08	(1.10)	0.14	(1.24)	0.21	(2.45)	0.21	(2.51)	-0.08	(2.51)	-0.08	(-0.92)
6	0.16	(1.55)	0.09	(1.16)	0.15	(1.25)	0.22	(2.39)	0.23	(2.44)	-0.05	(2.44)	-0.05	(-0.74)
7	0.17	(1.56)	0.09	(1.22)	0.16	(1.30)	0.24	(2.37)	0.25	(2.40)	-0.02	(2.40)	-0.02	(-0.36)
8	0.18	(1.56)	0.09	(1.21)	0.17	(1.39)	0.26	(2.35)	0.26	(2.38)	0.00	(2.38)	0.00	(-0.05)
9	0.19	(1.56)	0.09	(1.20)	0.18	(1.50)	0.27	(2.36)	0.27	(2.38)	0.01	(2.38)	0.01	(0.19)
10	0.19	(1.56)	0.10	(1.19)	0.19	(1.58)	0.28	(2.37)	0.28	(2.39)	0.01	(2.39)	0.01	(0.25)
11	0.20	(1.56)	0.10	(1.09)	0.21	(1.70)	0.29	(2.38)	0.29	(2.40)	0.02	(2.40)	0.02	(0.49)
12	0.20	(1.53)	0.10	(1.03)	0.22	(1.79)	0.30	(2.38)	0.31	(2.42)	0.03	(2.42)	0.03	(0.68)

(continued)

Table 8. Continued

Forecast	Panel B: Control variables													
	TR (DRS)	<i>t</i>	TR (Infl.)	<i>t</i>	TR (LTY)	<i>t</i>	TR (LTRR)	<i>t</i>	TR (TS)	<i>t</i>	TR (TBR)	<i>t</i>	TR (NEE)	<i>t</i>
1	0.02	(0.37)	0.02	(0.34)	0.03	(0.52)	0.02	(0.30)	0.02	(0.34)	0.03	(0.50)	0.02	(0.44)
2	0.08	(1.37)	0.08	(1.51)	0.09	(1.52)	0.08	(1.32)	0.07	(1.27)	0.09	(1.54)	0.08	(1.50)
3	0.12	(2.28)	0.12	(2.38)	0.13	(2.45)	0.12	(2.25)	0.11	(2.17)	0.13	(2.48)	0.12	(2.50)
4	0.12	(2.24)	0.12	(2.24)	0.13	(2.41)	0.12	(2.21)	0.11	(2.13)	0.13	(2.44)	0.12	(2.50)
5	0.10	(2.14)	0.10	(2.04)	0.11	(2.30)	0.10	(2.07)	0.09	(1.99)	0.11	(2.32)	0.10	(2.19)
6	0.09	(1.99)	0.08	(1.86)	0.10	(2.11)	0.09	(1.90)	0.08	(1.76)	0.10	(2.11)	0.09	(1.83)
7	0.08	(1.65)	0.07	(1.51)	0.09	(1.73)	0.08	(1.54)	0.07	(1.42)	0.09	(1.72)	0.08	(1.47)
8	0.08	(1.64)	0.07	(1.44)	0.09	(1.69)	0.08	(1.54)	0.07	(1.34)	0.09	(1.67)	0.08	(1.42)
9	0.08	(1.43)	0.07	(1.27)	0.09	(1.45)	0.07	(1.32)	0.06	(1.16)	0.09	(1.46)	0.07	(1.22)
10	0.06	(1.04)	0.05	(0.89)	0.07	(1.09)	0.06	(0.94)	0.04	(0.79)	0.07	(1.10)	0.06	(0.89)
11	0.06	(0.85)	0.05	(0.73)	0.07	(0.93)	0.05	(0.75)	0.04	(0.61)	0.07	(0.93)	0.05	(0.72)
12	0.05	(0.78)	0.04	(0.63)	0.06	(0.87)	0.04	(0.67)	0.03	(0.52)	0.06	(0.88)	0.05	(0.65)

(continued)

Table 8. Continued

Forecast	DRS	Infl.	LTY	LTRR	TS	TBR	NEE	<i>t</i>
1	-0.05	-0.04	0.03	0.09	0.05	0.01	-0.01	(-0.25)
2	-0.04	0.00	0.06	0.09	0.06	0.03	-0.01	(-0.08)
3	-0.01	-0.01	0.08	0.07	0.05	0.04	0.01	(0.07)
4	-0.02	-0.02	0.08	0.09	0.06	0.05	0.01	(0.08)
5	-0.04	-0.04	0.10	0.13	0.08	0.05	0.00	(-0.01)
6	-0.04	-0.05	0.12	0.14	0.09	0.07	-0.01	(-0.07)
7	-0.03	-0.06	0.14	0.15	0.10	0.08	-0.01	(-0.08)
8	-0.01	-0.07	0.16	0.14	0.12	0.09	-0.01	(-0.08)
9	0.01	-0.07	0.18	0.13	0.13	0.10	-0.02	(-0.12)
10	0.03	-0.07	0.20	0.11	0.14	0.10	-0.02	(-0.14)
11	0.04	-0.07	0.21	0.11	0.15	0.11	-0.02	(-0.13)
12	0.04	-0.08	0.23	0.12	0.16	0.12	-0.03	(-0.14)

Notes: Returns of the CRSP value-weighted market index returns are predicted with the following regressions: $r_{i,t+h} = \alpha + \beta TR_t + \gamma r_{i,t-k} + \delta GW_t + u_t$, where TR indicates the Hellinger Tail Risk. Additional controls (GW) indicate the Goyal and Welch (2008) surveyed predictors: book-to-market (BM), dividend payout (DP), earnings price ratio (EPR), dividend price ratio (DPR), dividend yield (DY), stock variance (SV), default yield spread (DYS), default return spread (DRS), inflation (Infl.), long-term rate of returns (LTRR), term spread (TS), treasury bill rate (TBR), and net equity expansion (NEE). In both panels A and B, we present the estimated coefficients for the Hellinger Tail Risk in the upper part and the estimated coefficient for the additional control variables in the lower part. *t*-Statistics are calculated using Newey–West variance matrix with 2–24 lags for 1–12 forecasting horizon. Sample: July 1962 to December 2012.

reinforces the interpretation of our measure as tail risk since the default spread ought to widen in periods when tail risk is prevalent. It is therefore hard to capture each effect independently.

In Panel B of [Table 8](#), we consider another set of seven predictors used in [Welch and Goyal \(2008\)](#): default return spread, inflation, long-term yield, long-term rate of returns, term spread, treasury bill rate, and net equity expansion. Only the long-term rate of bond returns shows a strong and statistically significant predictive power, but tails risk remains as a significant predictor 3-to-6 months ahead.

Overall, we can conclude that tail risk has a significant predictive power in the short run even after controlling for the main predictors of future stock market returns.

3.2. Economic Predictability

We now explore the relationship of our tail-risk measure with the real economy. Recently, in the aftermath of the great recession, several papers put forward facts and patterns about economic uncertainty, its fluctuations during business cycles, and its empirical relation with micro and macro growth dispersion measures. While measures of volatility may capture uncertainty, it is often the case that big negative shocks play an important role in increasing economic uncertainty. This asymmetry occurs because good news seem to build up more gradually over a period of time. Therefore, tail-risk measures may help in measuring better the effects of economic uncertainty on macroeconomic variables such as output growth and employment. Both [Bloom \(2009\)](#) and [Bali, Brown, and Caglayan \(2014\)](#) propose measures of macroeconomic uncertainty based on volatility of macroeconomic variables. We have seen that our measure of tail risk has a correlation close to 0.5 with these two uncertainty indicators. We have also characterized the jumps in tail risk in [Figure 3](#) and they often correspond to jumps in uncertainty identified by [Bloom \(2009\)](#).¹⁹ In what follows we will follow the vector auto-regressive impulse response function approach of [Bloom \(2009\)](#) to distinguish between the effects of volatility and tail risk on macroeconomic aggregate variables.

3.2.1 Impulse response functions to tail-risk shocks

To assess whether uncertainty shocks influenced firm-level decisions, [Bloom \(2009\)](#) considered several variables in his VAR approach: stock market returns, volatility (either market variance or an indicator capturing the biggest volatility events), the Federal Funds rate, wages, inflation, hours, employment, and production in manufacturing, in this order. A main conclusion was that a rise in economic uncertainty reduces industrial production and employment in the short run. We argue that some of the uncertainty effect captured by [Bloom's](#) study might actually be coming from tail risk, in the same line as [Kelly and Jiang \(2014\)](#).²⁰ The main theoretical insight behind our argument comes from a firm's

¹⁹ [Bloom \(2009\)](#) identifies 17 uncertainty dates defined as events associated with stock market volatility in excess of 1.65 standard deviations above its trend.

²⁰ These authors use the tail-risk measure that we described earlier applied to the manufacturing firms in the sample. It is therefore a very specific measure that we want to compare to our general-purpose measure based on principal components. It is also instructive to see how our measure performs given the two very different properties of the two measures as illustrated in [Section 2.3](#).

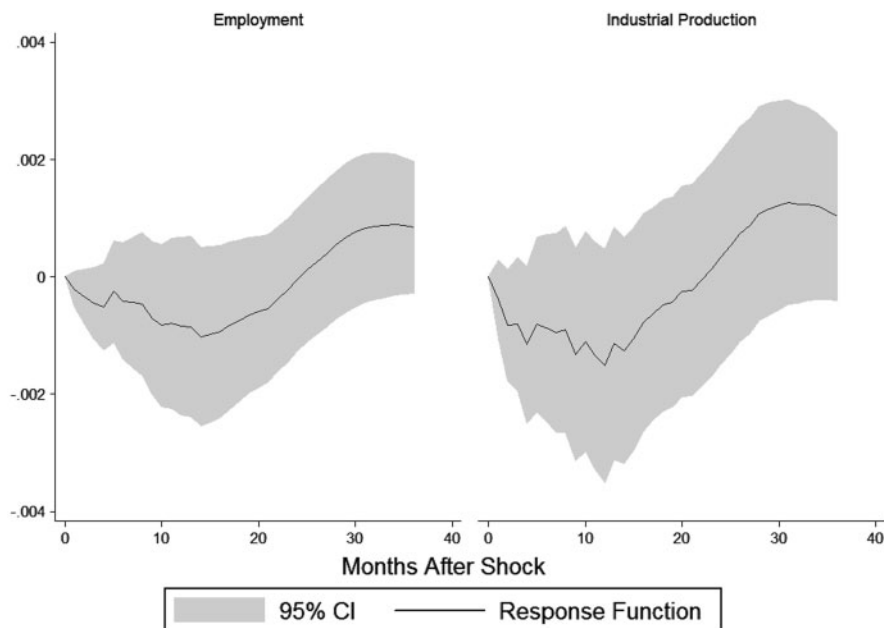


Figure 7. This figure presents the impulse response functions for Employment and Industrial Production following a shock in the Hellinger Tail Risk. Results follow from a vector auto-regressive approach similar to Bloom (2009) including an additional Tail-Risk series (still controlling for the market variance).

perspective on investment decisions in the presence of uncertainty. If firms look at their investment choices as real options, an increase in uncertainty may defer her decision to invest, to produce, or to hire. Therefore, a rise in volatility or tail risk may affect aggregate production or employment as well as investment.²¹

Therefore, we expand Bloom (2009)'s vector auto-regressive approach by adding our measure of tail risk to the set of variables considered.²² In terms of order we include it just after volatility. In Figure 7, we plot the impulse response functions on both employment and industrial production to a one standard deviation shock in the monthly estimated tail-risk measure.²³ The gray area represents the 95% confidence interval for the response function while the solid line represents the response itself. In Figure 8, we plot the same response functions for a one standard deviation shock to volatility measured by stock market variance.²⁴ For the shocks to tail risk, the effect is less abrupt but it seems to be followed by

- 21 Kellogg (2014) and Jurado, Ludvigson, and Ng (2015) explore in more depth this particular channel.
- 22 To be precise, market returns measured by S&P 500 returns lagged, wages, hours, industrial production, and employment are in logs, inflation is measured as the difference in log CPI. The variables are in the order indicated in the previous paragraph.
- 23 The only difference between our approach and Bloom's one is that we perform the impulse response analysis with an additional tail-risk factor.
- 24 Bloom (2009) uses realized variance before the availability of VXO and VXO afterwards.



Figure 8. This figure presents the impulse response functions for Employment and Industrial Production following a shock in the market variance. Results follow from a vector auto-regressive approach similar to Bloom (2009) including an additional Tail-Risk series.

a more positive recovery compared with a volatility shock where after 10 months the strong negative shock has been absorbed without any ensuing significant positive effect. Although the evidence is not strong, it seems that a tail-risk shock has a recovery period after the initial negative shock. In Figures 9 and 10, we plot the same respective impulse response functions but this time with Bloom (2009) indicator of big volatility events. Although the shapes of the reaction functions are similar, the confidence intervals are now more telling about the effects of shocks to volatility and tail risk. The responses to tail risk are of greater magnitude, distinctively negative in the short run up to 10–15 months, and then becoming significantly positive at an horizon of 30–35 months. For shocks to the volatility indicator, small initial negative effects die after 5 months and never pick up significantly afterwards for employment. However, for industrial production, after a small contraction in the first 3 months, there is a small rebound at an horizon of 10–15 months. These results tend to show that it is difficult to disentangle uncertainty created by big events from the uncertainty associated with more volatile environments.

3.2.2 Business cycles

Bloom (2009) states that almost every macroeconomic indicator of uncertainty appears to be countercyclical. In Figure 3, we plot our tail risk, in blue, and highlight with shades of gray the NBER recession periods. To give a better idea of the relation between tail risk and recessions we also plot in red the same tail-risk measure after passing the series through a Hodrick–Prescott filter to isolate the cyclical component. This makes clear that usually tail risk is higher whenever the economy is in a recession. Therefore, a natural question to ask

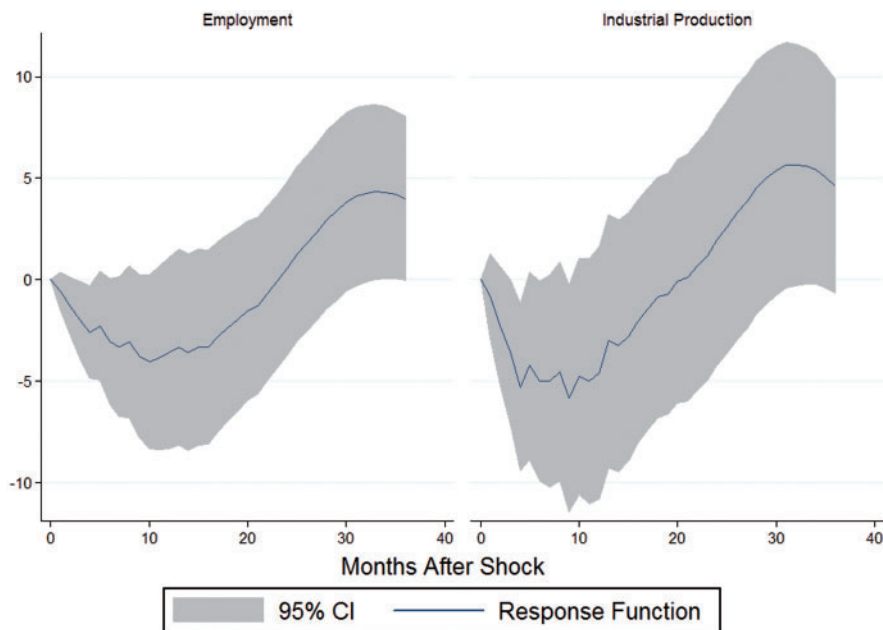


Figure 9. This figure presents the impulse response functions for Employment and Industrial Production following a shock in the Hellinger Tail Risk. Results follow from a vector auto-regressive approach similar to Bloom (2009) including an additional Tail-Risk series (still controlling for the big volatility events indicator).

is whether our measure of tail risk has some predictive power over economic downturns. Our approach is comparable to Allen, Bali, and Tang (2012) who consider measures of systemic risk for the financial sector as predictors for economic downturns. While they focus their argument on the special intermediation role of the financial industry, our measure is economy-wide since it is based on portfolios of all firms making up the stock market. Therefore, in the same logic evoked in the previous section our tail risk should anticipate movements in the real activity or business cycle indicators.

We collect a large set of macroeconomic indicators and assess their predictability by our tail-risk index at monthly horizons between 1 and 12 months. Overall, we have eight indices that are available for different samples: 1) ADS, the Aruoba, Diebold and Scotti macroeconomic activity indicator (02/1960–04/2014); 2) KCFED, the Kansas City FED macroeconomic indicator index (01/1990–04/2014); the NBER recession indicator (a recession period dummy, 07/1926–04/2014); CFNAI, the Chicago FED National Activity Index (02/1967–04/2014); the financial and macroeconomic indexes of Jurando, Ludvigson, and Ng (2015) (07/1960–04/2014); the St. Louis FED Financial Stress Index (01/1994–04/2014); and EPU, the Economic Policy Uncertainty Index of Backer, Bloom, and Davis (2015) (01/1985–04/2014). We run the following regression:

$$I_{t+i} = \beta_0 + \beta \text{TR}_t + \sum_{k=0}^{11} \gamma_i I_{t-k} + \epsilon_t, \quad (20)$$

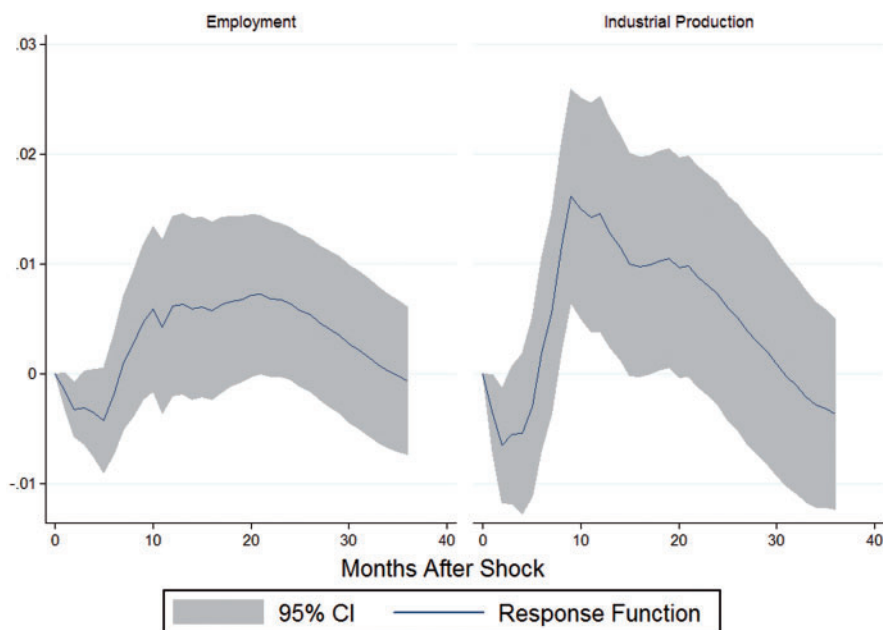


Figure 10. This figure presents the impulse response functions for Employment and Industrial Production following a shock in the big volatility event indicator. Results follow from a vector autoregressive approach similar to Bloom (2009) including an additional Tail-Risk series.

where I stands for the activity index we consider and i indexes the forecasting horizon ($i = 1, \dots, 12$). Note that since the NBER recession indicator is a binary variable we estimate a Probit model. Regression results are reported in Table 9. We observe that for all indicators except the KCFED there is a significant predictability at short horizons up to 3–4 months ahead. For the NBER and the financial uncertainty indicator the predictive power is strong and extends to all horizons up to 12 months ahead. The signs are of course negative to capture the negative effect of current tail-risk increases on future activity, except of course for NBER recessions and EPU which are downturn and uncertainty measures, respectively.

To conclude this section we discuss the inter-temporal relationship between our tail-risk measure and four other tail-risk measures over different time periods,²⁵ namely the Bali, Brown, and Caglayan (2014) macroeconomic uncertainty index (01/1994–12/2013); the BTX tail-risk index (01/19963–08/2013) in Bollerslev, Todorov, and Xu (2015); the Kelly and Jiang (2014) tail-risk measure (01/1963–12/2010); and finally the Allen, Bali, and Tang (2012) systemic risk measure (01/1973–12/2012) called CATFIN. We use the same prediction regression framework as in Equation (20). In Table 10, it can be seen that we predict well the Macro and KJ indicators at all horizons. For the BTX index, the predictability is limited to the very short horizons, while for CATFIN we see some predictability in

25 These time periods are determined by the currently available series on the web sites of the researchers.

Table 9. Macroeconomic activity prediction regressions

Panel A													
Forecast	ADS	<i>t</i>	R ²	KCFED	<i>t</i>	R ²	NBER	<i>t</i>	R ²	CFNAI	<i>t</i>	R ²	R ²
1	-0.05	(-2.70)	0.84	0.01	(-0.27)	0.92	0.69	-3.17	0.78	-0.10	(-2.80)	0.53	0.53
2	-0.09	(-3.09)	0.52	-0.03	(-0.55)	0.81	0.52	-2.50	0.62	-0.12	(-3.37)	0.46	0.46
3	-0.08	(-2.40)	0.39	-0.09	(-1.05)	0.74	0.58	-3.27	0.50	-0.11	(-2.97)	0.36	0.36
4	-0.05	(-1.57)	0.30	-0.16	(-1.39)	0.65	0.54	-3.45	0.40	-0.08	(-1.89)	0.27	0.27
5	-0.04	(-1.27)	0.22	-0.17	(-1.40)	0.56	0.45	-3.24	0.30	-0.06	(-1.68)	0.19	0.19
6	-0.03	(-0.76)	0.17	-0.20	(-1.46)	0.49	0.43	-3.27	0.22	-0.07	(-1.89)	0.16	0.16
7	-0.02	(-0.45)	0.14	-0.21	(-1.36)	0.41	0.45	-3.65	0.15	-0.03	(-0.87)	0.12	0.12
8	-0.03	(-0.65)	0.12	-0.18	(-1.23)	0.33	0.42	-3.56	0.10	-0.03	(-0.70)	0.10	0.10
9	-0.03	(-0.57)	0.10	-0.16	(-1.15)	0.29	0.38	-3.35	0.07	-0.07	(-1.66)	0.09	0.09
10	-0.02	(-0.39)	0.08	-0.18	(-1.28)	0.26	0.40	-3.57	0.05	-0.05	(-1.26)	0.06	0.06
11	-0.04	(-0.79)	0.06	-0.12	(-1.03)	0.22	0.43	-3.80	0.03	-0.07	(-1.37)	0.05	0.05
12	-0.05	(-0.96)	0.04	-0.07	(-0.70)	0.19	0.42	-3.74	0.03	-0.10	(-1.69)	0.04	0.04

(continued)

Table 9. Continued

Panel B												
Forecast	Fin. Uncert.	<i>t</i>	R ²	Macro Uncert.	<i>t</i>	R ²	St. Louis	<i>t</i>	R ²	EPU	<i>t</i>	R ²
1	-0.22	(-2.44)	0.32	-0.10	(-2.35)	0.42	-0.17	(-1.81)	0.15	0.08	(4.18)	0.75
2	-0.26	(-4.17)	0.14	-0.16	(-2.59)	0.14	-0.25	(-2.21)	0.10	0.09	(3.80)	0.61
3	-0.20	(-4.47)	0.07	-0.14	(-2.17)	0.07	-0.24	(-2.51)	0.08	0.11	(3.15)	0.56
4	-0.16	(-3.40)	0.05	-0.17	(-2.16)	0.04	-0.25	(-2.47)	0.09	0.03	(0.82)	0.52
5	-0.14	(-2.66)	0.04	-0.13	(-1.66)	0.03	-0.17	(-1.63)	0.06	0.05	(1.05)	0.48
6	-0.12	(-2.63)	0.03	-0.12	(-1.83)	0.03	-0.14	(-1.79)	0.06	0.04	(0.90)	0.44
7	-0.13	(-2.66)	0.03	-0.13	(-2.16)	0.03	-0.16	(-1.80)	0.06	0.09	(2.24)	0.43
8	-0.16	(-3.38)	0.04	-0.11	(-1.90)	0.03	-0.04	(-0.55)	0.05	0.04	(1.05)	0.42
9	-0.11	(-2.83)	0.02	-0.11	(-2.31)	0.04	-0.08	(-1.07)	0.05	0.05	(0.86)	0.41
10	-0.10	(-3.05)	0.02	-0.09	(-1.90)	0.03	-0.14	(-2.23)	0.05	0.06	(1.05)	0.40
11	-0.15	(-3.06)	0.03	-0.08	(-2.18)	0.03	-0.02	(-0.35)	0.03	0.11	(1.75)	0.38
12	-0.07	(-1.81)	0.02	-0.05	(-1.52)	0.03	0.02	(0.59)	0.04	0.13	(2.43)	0.34

Notes: Predictive regressions are run for a variety of macroeconomic indicators (estimating sample in parentheses). ADS represents the Aruoba, Diebold, and Scotti macroeconomic activity indicator (02/1960–04/2014); KCFED represents the Kansas City FED macroeconomic indicator index (01/1990–04/2014); NBER represents a recession period dummy (07/1926–04/2014); CFNAI represents the Chicago FED National Activity Index (02/1967–04/2014); Fin and Macro Uncert. indicates Jurando, Ludvigson, and Ng (2015) financial and macroeconomic uncertainty index, respectively (07/1960–04/2014); St. Louis stands for the St. Louis FED Financial Stress Index (01/1994–04/2014); and EPU represents the Economic Policy Uncertainty Index of Baker, Bloom, and Davis (2015) (01/1985–04/2014). All regressions control for 12 lags of the endogenous variable. All *t*-statistics are computed with a 24-lag Newey–West correction. For the NBER variable, we report the Probit regression results.

Table 10. Prediction of other tail-risk measures

Forecast	Macro	t	R^2	BTX	t	R^2	KJ	t	R^2	CATFIN	t	R^2
1.00	0.04	(-1.88)	0.96	0.32	(-2.14)	0.48	0.02	(-1.17)	0.88	-0.01	(-0.33)	0.54
2.00	0.11	(-1.80)	0.92	0.18	(-2.94)	0.30	0.03	(-1.33)	0.82	-0.05	(-1.87)	0.43
3.00	0.18	(-2.18)	0.88	0.15	(-1.18)	0.15	0.04	(-2.05)	0.80	-0.07	(-1.62)	0.39
4.00	0.18	(-2.40)	0.83	0.08	(-1.90)	0.11	0.05	(-2.11)	0.77	-0.06	(-1.41)	0.34
5.00	0.18	(-2.83)	0.77	0.14	(-1.34)	0.10	0.05	(-1.82)	0.74	-0.14	(-3.13)	0.34
6.00	0.17	(-3.11)	0.71	0.08	(-1.46)	0.06	0.08	(-2.69)	0.72	-0.12	(-2.06)	0.30
7.00	0.17	(-3.41)	0.65	0.20	(-2.85)	0.08	0.09	(-2.67)	0.69	-0.09	(-1.36)	0.28
8.00	0.15	(-3.15)	0.59	0.09	(-1.14)	0.04	0.08	(-2.45)	0.66	-0.06	(-0.91)	0.27
9.00	0.15	(-2.98)	0.53	0.11	(-1.56)	0.05	0.08	(-2.30)	0.65	-0.04	(-0.75)	0.24
10.00	0.14	(-2.62)	0.46	0.13	(-1.84)	0.05	0.09	(-2.29)	0.62	-0.03	(-0.63)	0.22
11.00	0.14	(-2.77)	0.39	0.08	(-0.99)	0.04	0.13	(-2.88)	0.61	-0.03	(-0.50)	0.20
12.00	0.15	(-2.82)	0.34	0.15	(-1.65)	0.05	0.15	(-3.38)	0.59	-0.04	(-0.85)	0.17

Notes: Predictive regressions are run for a variety of Tail-Risk indexes using the Hellinger Tail Risk as a predictor. KJ refers to Kelly and Jiang (2014) tail-risk index (01/1963–12/2010); BTX refers to Bollerslev, Todorov, and Xu (2015) tail-risk index (01/19963–08/2013); Macro refers to Bali, Brown, and Caglayan (2014) macro-economic uncertainty index (01/1994–12/2013); and CAFTIN denotes Allen, Bali, and Tang (2012) systemic risk measure (01/1973–12/2012). All regressions control for 12 lags of the endogenous variable. Bali, Brown, and Caglayan (2014) is I(1) so we perform the regression in first differences. All t statistics are computed with a 24-lag Newey–West correction.

the first 6 months. We also note that the relationship with Bollerslev, Todorov, and Xu (2015) is almost monotonically decreasing, but that it is the contrary for the Kelly and Jiang (2014) measure. This is consistent with the fact that the KJ tail risk is a slow moving, very persistent measure. On the contrary, the BTX measure of Bollerslev, Todorov, and Xu (2015), like ours, is very volatile and quickly mean reverts.

4 Robustness

To build our measure of tail risk, we have made several choices regarding the parameter of the Cressie-Read discrepancy family ($\gamma = -0.5$, so-called Hellinger discrepancy), the number of principal components of the 25 size and book-to-market portfolios, the number of days over which the excess ES is computed (30 days), and the threshold of the shortfall set at 10%. Therefore, we will first verify whether our results are sensitive to these choices. Second, we argued that there were good reasons for choosing an ES measure over a VaR measure. However, other papers have built tail or downside risk VaR-based measures and shown that they were superior to other risk measures (see in particular Bali, Demirtas, and Levy, 2009). We will then reproduce our results for VaR measures and compare them to our original results with ES. Finally, our benchmark measure has been obtained from a set of five principal components of the 25 size and book-to-market portfolios. We already checked that the sorting portfolio results survived when we extracted the principal components from the whole cross-section of individual stocks. Another important question concerns the choice of portfolios. Will the measure keep similar properties when constructed from industry portfolios, or from the financial sector only, for example? This will be our third line of investigation for measuring the robustness of our methodology for measuring tail risk.

Table 11. Robustness: sorted portfolio

	Panel A: One month holding period							
	$\gamma = -3$	$\gamma = -1$	$\gamma = 0$	$\gamma = 1$	10 PC	45 D.	20 D.	5%
Average return	-1.55	-1.62	-1.42	-1.37	-1.28	-1.68	-1.29	-1.52
	-3.46	-3.61	-3.32	-3.16	-2.86	-3.68	-2.92	-3.34
FF3	-1.17	-1.24	-1.10	-1.05	-0.91	-1.19	-0.79	-0.99
	-3.08	-3.35	-3.01	-2.73	-2.37	-3.02	-2.20	-2.61
FF3+MOM	-1.22	-1.29	-1.15	-1.07	-0.93	-1.19	-0.86	-1.03
	-3.12	-3.36	-3.02	-2.71	-2.38	-2.96	-2.32	-2.63
FF3+MOM+LIQ	-1.29	-1.39	-1.21	-1.13	-1.02	-1.29	-0.98	-1.13
	-2.82	-3.04	-2.67	-2.41	-2.20	-2.72	-2.18	-2.44
FF3+MOM+LIQ+VOL	-0.83	-0.96	-0.74	-0.60	-0.30	-0.53	-0.57	-0.68
	-1.79	-2.09	-1.62	-1.36	-0.70	-1.23	-1.26	-1.63
	Panel B: One year holding period							
Average return	-14.67	-14.60	-12.27	-12.43	-9.42	-15.57	-11.32	-14.71
	-3.06	-3.23	-2.88	-3.02	-2.41	-3.54	-2.51	-3.63
FF3	-3.07	-3.78	-2.17	-2.19	0.56	-3.63	-0.29	-4.18
	-1.01	-1.38	-0.82	-0.84	0.22	-1.33	-0.10	-1.55
FF3+MOM	-5.50	-5.68	-4.08	-3.30	-1.61	-4.54	-2.42	-4.88
	-2.02	-2.29	-1.71	-1.40	-0.68	-1.87	-0.97	-1.94
FF3+MOM+LIQ	-6.10	-5.82	-3.76	-3.09	-2.05	-4.82	-2.67	-6.76
	-2.24	-2.40	-1.65	-1.29	-0.90	-1.88	-1.18	-2.79
FF3+MOM+LIQ+VOL	9.60	10.06	13.37	13.27	8.67	8.63	17.32	8.13
	0.80	0.91	1.20	1.16	0.92	0.67	1.38	0.84

Notes: Returns are reported for decile portfolios of CRSP stocks with code 10-11 sorted according to their tail-risk hedging capacity. The columns indicate differences between the tail-risk measure considered and the baseline case ($\gamma = -0.5$, 30 days returns, five principal components, and 10% VaR threshold). PC denotes the number of principal components used, D the number of days, and 5% the VaR threshold. For each month in our sample from February 1967 to December 2013, we sort the stocks and track their returns one-month post formation (Panel A) or one-year post formation (Panel B). In the first line we report the average portfolio returns. In the following lines we report the α 's of regressions where we control for the factors indicated in the first column. We only report the values for the high-minus-low portfolios. Newey–West t -statistics reported between parentheses are computed with one lag for monthly results and 12 lags for yearly results.

4.1. Robustness to the Parameters of the Measure

In Table 11, we report the returns of the Long-Short portfolios associated with different discrepancy parameters (first four columns). We can see that the patterns and magnitudes of returns are very similar to the benchmark results. The statistical significance of the last set of control variables, which includes volatility, disappears for 3 out of 4. The discrepancy closest to ours in the negatives, $\gamma = -1$, provides therefore the most similar results. Adding principal components (10 instead of 5) does not improve results, nor increasing nor reducing the number of days over which we compute the ES. Again, we seem to lose precision in estimating the risk premiums, especially after controlling for volatility. Finally, setting the

threshold at 5% instead of 10% does not have much of an effect. We have also computed the sensitivity of our prediction results to these different values of the parameters.

For both market returns prediction and macroeconomic indicators forecast we arrived at very similar results for all configurations and all variables. We can then confidently conclude that the results are very robust to the parameters used to construct our tail-risk measure. One could add that they are too robust in the sense that the risk neutralization could be done indifferently with any gamma value. In this regard it is important to emphasize that our performance criterion is a simple average where the timing of the prediction is not taken into account. If we consider that errors are more costly in bad times, then more important differences will appear between the various discrepancy gamma parameters. Risk neutralization will matter more.

4.2. Measuring Tail Risk from Industry and Sector Portfolios

We chose to build our benchmark tail-risk measure on the first five principal components of 25 size and book-to-market portfolios. In this section, we want to illustrate that our results in terms of risk premia and predictability are not changed significantly if we take other reference portfolios. We have seen in Section 2.3 that risk premia on the high-minus-low portfolios were similar when the five principal components were computed from the whole set of securities available in the cross-section. Here, we choose three sets of portfolios: a set of 10 industry portfolios, an aggregate portfolio of the financial sector, and an aggregate portfolio of the real sector. Results are reported in Table 12. Apart from the sorting portfolios columns where alphas are reported for the high-minus-low portfolio we also feature market returns predictions, in a long sample [similar to Kelly and Jiang (2014) starting in the 60s] and a short one [as in Bollerslev, Todorov, and Xu (2015) starting in the 90s]. For the long sample we also show market prediction results controlling for dividend yield and stock variance as predictors.

In the first panel featuring 10 industry portfolios, risk premiums are remarkably similar in terms of magnitudes and statistical significance to our benchmark estimates in Table 4. Note that the sign of the one-year holding period when controlling for all factors including volatility is now negative, although not significant. For the prediction of market returns, the statistical significance patterns are very similar, where the short-term prevails, but the regression coefficients are lower in general.

In the financial sector panel, risk premia are generally higher in absolute value, especially for the long holding period, but the statistical significance remains quite comparable. The predictive power of market returns disappears in the short sample (after the 90s) which may be explained by the turmoil in financial markets during this period. We also lose some significance and impact when controlling for stock variance. This is to be expected since tail-risk measured from financial returns is harder to disentangle from the high stock variance episodes. This interpretation is supported by the evidence in the last panel when we consider tail risk measured from the real sector alone.

The risk premia are again very close to the benchmark values in Table 4 with a slightly higher level of precision in the estimates. The predictive power is also estimated with much more precision, especially when controlling for other predictors.

We did not include results on economic predictability because they were very similar to the benchmark results with the size and book-to-market portfolios.

Table 12. Continued

10 industry portfolios													
Period	Return prediction			Tail-risk prediction			Sorting portfolios						
	TR-KJ	<i>t</i>	TR-BTX	<i>t</i>	KJ	BTX	<i>t</i>	FF3	FF3+MOM	FF3+MOM+LIQ	FF3+MOM+LIQ+VOL	α	<i>t</i>
9	0.09	(1.15)	0.20	(1.39)	0.11	(2.08)	0.05	(0.77)	FF3			-7.45	(-2.19)
10	0.09	(1.07)	0.19	(1.27)	0.10	(1.63)	0.03	(0.42)	FF3+MOM			-9.39	(-3.02)
11	0.08	(0.94)	0.17	(1.12)	0.10	(1.55)	0.02	(0.31)	FF3+MOM+LIQ			-10.67	(-3.32)
12	0.07	(0.87)	0.17	(1.16)	0.09	(1.37)	0.00	(0.06)	FF3+MOM+LIQ+VOL			9.87	(0.87)
Real sector tail risk													
1	0.11	(3.39)	0.08	(1.30)	0.06	(1.35)	0.15	(3.60)	Average	One month		-1.57	(-3.73)
2	0.14	(3.94)	0.11	(1.60)	0.13	(2.90)	0.19	(4.85)	FF3			-1.16	(-3.27)
3	0.16	(4.17)	0.16	(2.29)	0.18	(3.91)	0.23	(5.24)	FF3+MOM			-1.21	(-3.30)
4	0.13	(3.61)	0.15	(2.27)	0.17	(3.41)	0.20	(4.44)	FF3+MOM+LIQ			-1.31	(-2.99)
5	0.13	(3.40)	0.17	(3.14)	0.13	(2.62)	0.14	(3.31)	FF3+MOM+LIQ+VOL			-0.90	(-2.19)
6	0.11	(2.87)	0.15	(2.53)	0.13	(2.41)	0.13	(3.09)	Average	One year		-14.15	(-3.54)
7	0.11	(2.91)	0.15	(2.34)	0.12	(2.36)	0.12	(2.82)	FF3			-4.16	(-1.72)
8	0.08	(2.21)	0.10	(1.45)	0.12	(2.45)	0.11	(2.52)	FF3+MOM			-4.66	(-2.12)
9	0.06	(1.47)	0.07	(0.98)	0.11	(2.19)	0.08	(1.94)	FF3+MOM+LIQ			-5.61	(-2.51)
10	0.06	(1.42)	0.08	(1.10)	0.08	(1.86)	0.05	(1.28)	FF3+MOM+LIQ+VOL			7.62	(0.80)
11	0.04	(1.06)	0.06	(0.88)	0.08	(1.94)	0.05	(1.39)					
12	0.04	(1.04)	0.07	(0.91)	0.07	(1.65)	0.04	(0.87)					

Notes: Assorted robustness tests are run for a tail-risk measure constructed using industry portfolios. Columns 2–5 feature coefficients and *t*-stats of predictive regressions for market returns for two estimating samples, the first one corresponding to Kelly and Jiang (2015, KJ) and the second one to Bollerstev, Todorov, and Xu (2015, BTX). In Columns 6–9 we report the coefficients of predictive regressions of the KJ and BTX measures by our tail-risk measure. Returns for the high-minus-low sorted portfolios are reported in Columns 10–12. All *t*-statistics are computed using Newey–West standard errors.

Table 13. Robustness: VaR tail risk

Period	Return prediction			Tail-risk prediction			Sorting portfolios			
	TR-KJ	TR-BTX	t	KJ	t	BTX	t	α	t	
1	0.05	-0.01	(0.76)	-0.01	(-0.32)	0.27	(1.36)	One month	-1.17	(-2.58)
2	0.06	-0.02	(0.80)	0.01	(0.26)	0.12	(0.69)	Average	-0.86	(-1.96)
3	0.08	0.03	(1.08)	0.02	(0.76)	0.22	(0.91)	FF3	-0.84	(-1.89)
4	0.09	0.08	(1.26)	0.03	(0.99)	0.11	(0.75)	FF3+MOM	-0.84	(-1.65)
5	0.09	0.11	(1.29)	0.04	(0.97)	0.15	(0.84)	FF3+MOM+LIQ	0.06	(0.13)
6	0.09	0.12	(1.18)	0.05	(1.30)	0.14	(0.77)	FF3+MOM+LIQ+VOL		
7	0.09	0.15	(1.10)	0.07	(1.51)	0.15	(0.75)	One year		
8	0.07	0.15	(0.79)	0.08	(1.64)	0.17	(0.67)	Average	-10.97	(-3.24)
9	0.07	0.13	(0.73)	0.09	(1.83)	0.13	(0.70)	FF3	-2.16	(-0.77)
10	0.07	0.15	(0.75)	0.10	(1.78)	0.17	(0.87)	FF3+MOM	-2.10	(-0.95)
11	0.06	0.19	(0.62)	0.14	(2.45)	0.16	(0.85)	FF3+MOM+LIQ	-1.83	(-0.75)
12	0.05	0.18	(0.50)	0.16	(2.86)	0.17	(1.20)	FF3+MOM+LIQ+VOL	18.05	(1.63)

Notes: Assorted robustness tests are run for a risk-neutral VaR measure. Columns 2–5 feature coefficients and *t*-stats of predictive regressions for market returns for two estimating samples, the first one corresponding to Kelly and Jiang (2015, KJ) and the second one to Bollerslev, Todorov, and Xu (2015, BTX). In Columns 6–9 we report the coefficients of predictive regressions of the KJ and BTX measures by our tail-risk measure. Returns for the high-minus-low sorted portfolios are reported in Columns 10–12. All *t*-statistics are computed using Newey–West standard errors.

We can conclude that the risk-neutralized excess ES is a very robust measure of tail risk once we aggregate the individual stock returns into portfolios irrespective of the nature of the portfolios. Risk premia are of the same magnitude and estimated roughly with the same degree of precision. The financial sector tail-risk measure tends to accentuate the premia but the pattern remains the same.

4.3. Measuring Tail Risk from Risk-Neutral VaR

We have argued that ES should be preferred to VaR on theoretical grounds, but some researchers have produced some empirical results that support the opposite. Therefore, we construct a tail-risk measure based on the risk-neutral VaR that we used in computing our excess ES for the base case with the five principal components of the 25 size and book-to-market portfolios. We report some selected representative results in Table 13. Let us start with our benchmark risk premium measure for the high-minus-low portfolio in the last columns of the table. We can see right away that the only significant figures are the averages when we do not control for other factors. Although the signs are preserved compared with our base-case results in Table 4 when we control for other factors, the magnitudes are different and the statistical significance is either marginal or disappears.

The other columns illustrate also the poor performance of the risk-neutral VaR tail risk. Market returns are not predicted well, neither in a long sample nor in a short one. No coefficient is nearly significant. The same is true for the prediction of the Kelly and Jiang (2014) tail risk or the option-based Bollerslev, Todorov, and Xu (2015) measure. These series were predicted very well by the Hellinger excess ES measure, especially the Kelly Jiang measure. This lack of predictive power extends to all series, except the macroeconomic uncertainty index and the NBER indicator of recessions. Finally, impulse response functions for employment and industrial production (not reported for space considerations) are essentially zero.

5 Conclusion

We propose a new measure of tail risk based on the risk-neutralized returns of a few principal components of the cross-section of stock returns. We have shown that the results are robust whether we compute these principal components from a set of aggregate portfolios (Fama–French size and book-to-market portfolios or industry portfolios) or from the whole cross-section of CRSP returns. This measure has two main advantages: it accounts for changes in economic conditions through the risk-neutralization nonparametric procedure and it is based only on stock returns. Not using option prices is useful to extend the sample considerably but opens also the possibility to apply the measure in markets where liquid option markets do not exist. Moreover, a similar measure can be computed for many other asset markets.

The series computed from 1926 to the present captures all main events that have increased uncertainty in the economy, whether they find their source in the financial market or in the political arena. It is therefore a tail-risk measure that goes beyond financial systemic risk. We have investigated thoroughly its relation with other tail-risk measures. For the option-based measures, our empirical comparison revealed that the implications of both measures were very similar in terms of risk premia for short and long holding periods. However, we have shown that the nature of our series is very different from the tail-risk

measure proposed by Kelly and Jiang (2014). The latter is very persistent and does not capture the big uncertainty events. In terms of forecasting power of market returns our series has a good performance in the short-run while the Kelly and Jiang (2014) tail risk predicts well 6–12 months ahead.

Our extensive investigation of economic predictability has confirmed the usefulness of our measure for capturing future conditions in employment, output, or more comprehensive measures of real activity. A detailed comparison with the Bloom (2009) indicator of economic uncertainty based on volatility has shown that our tail-risk measure adds new information and implies different impulse response functions for employment and industrial production. Finally, our measure predicts remarkably well the NBER recessions up to 12 months ahead and the data-rich aggregate financial and macroeconomic indicators of economic uncertainty produced recently by Jurado, Ludvigson, and Ng (2015). Our limited-information measure appears to be a very useful indicator to test theories of business cycles with economic shocks or to estimate or calibrate equilibrium asset pricing models with disasters, ambiguity aversion, or disappointment aversion, all implying an important role for tail events.

References

- Adrian, T., and M. K. Brunnermeier. 2014. "Covar." Working paper, NY FED and Princeton University.
- Agarwal, V., and N. Y. Naik. 2004. Risks and Portfolio Decisions Involving Hedge Funds. *Review of Financial Studies* 17: 63–98.
- Ait-Sahalia, Y., and A. W. Lo. 1998. Nonparametric Estimation of State-Price Densities Implicit in Financial Asset Prices. *Journal of Finance* 53: 499–547.
- Ait-Sahalia, Y., and A. W. Lo. 2000. Nonparametric Risk Management and Implied Risk Aversion. *Journal of Econometrics* 94: 9–51.
- Allen, L., T. G. Bali, and Y. Tang. 2012. Does Systemic Risk in the Financial Sector Predict Future Economic Downturns? *Review of Financial Studies* 25: 3000–3036.
- Almeida, C., K. Ardison, and R. Garcia. 2016. "High Frequency Tail Risk." Working paper, EPGE/FGV and EDHEC Business School.
- Almeida, C., and R. Garcia. 2012. Assessing Misspecified Asset Pricing Models with Empirical Likelihood Estimators. *Journal of Econometrics* 170: 519–537.
- Almeida, C., and R. Garcia. 2016. Economic Implications of Nonlinear Pricing Kernels. *Management Science*, forthcoming.
- Artzner, P., F. Delbaen, J. Eber, and D. Heath. 1999. Coherent Measures of Risk. *Mathematical Finance* 9: 223–228.
- Backus, D., M. Chernov, and S. Zin. 2014. Sources of Entropy in Representative Agent Models. *Journal of Finance* 69: 51–99.
- Bali, T. G., S. J. Brown, and M. O. Caglayan. 2014. Macroeconomic Risk and Hedge Fund Returns. *Journal of Financial Economics* 114: 1–19.
- Bali, T. G., N. Cakici, and F. Chabi-Yo. 2011. A Generalized Measure of Riskiness. *Management Science* 57: 1406–1423.
- Bali, T. G., N. Cakici, and R. F. Whitelaw. 2014. Hybrid Tail Risk and Expected Stock Returns: When Does the Tail Wag the Dog? *Review of Asset Pricing Studies* 4: 206–246.
- Bali, T. G., K. O. Demirtas, and H. Levy. 2009. Is there a Relation between Downside Risk and Expected Stock Returns? *Journal of Financial and Quantitative Analysis* 44: 883–909.
- Basak, S., and A. Shapiro. 2001. Value-at-Risk-Based Risk Management: Optimal Policies and Asset Prices. *The Review of Financial Studies* 14: 371–405.

- Bates, D. S. 1991. The Crash of '87: Was It Expected? The Evidence from Options Markets. *Journal of Finance* 46: 1009–1044.
- Bloom, N. 2009. The Impact of Uncertainty Shocks. *Econometrica* 77: 623–685.
- Bollerslev, T., and V. Todorov. 2011. Tails, Fears, and Risk Premia. *Journal of Finance* 66: 2165–2211.
- Bollerslev, T., V. Todorov, and L. Xu. 2015. Tail Risk Premia and Return Predictability. *Journal of Financial Economics* 118: 113–134.
- Breeden, D. T., and R. H. Litzenberger. 1978. Prices of State-Contingent Claims Implicit in Option Prices. *The Journal of Business* 51: 621–651.
- Brownlees, C., and R. Engle. 2015. “SRISK: A Conditional Capital Shortfall Measure of Systemic Risk.” Working paper, Universitat Pompeu Fabra and NYU Stern.
- Duffie, D. 1996. *Dynamic asset pricing theory*, 2nd edn. Princeton, NJ: Princeton University Press.
- Farago, A., and R. Tedongap. 2015. “Downside Risks and the Cross-Section of Asset Returns.” Working paper, Stockholm School of Economics and ESSEC Business School.
- Gospodinov, N., R. Kan, and C. Robotti. 2016. On the Properties of the Constrained Hansen–Jagannathan Distance. *Journal of Empirical Finance*, 36: 121–150.
- Hansen, L. P., and R. Jagannathan. 1991. Implications of Security Market Data for Models of Dynamic Economies. *Journal of Political Economy* 99: 225–262.
- Jurado, K., S. C. Ludvigson, and S. Ng. 2015. Measuring Uncertainty. *American Economic Review* 105: 1177–1216.
- Kellogg, R. 2014. The Effect of Uncertainty on Investment: Evidence from Texas Oil Drilling. *American Economic Review* 104: 1698–1734.
- Kelly, B., and H. Jiang. 2014. Tail Risk and Asset Prices. *Review of Financial Studies* 27: 2841–2871.
- Kitamura, Y., T. Otsu, and K. Evdokmov. 2013. Robustness, Infinitesimal Neighborhoods, and Moment Restrictions. *Econometrica* 81: 1185–1201.
- Kozak, S., S. Nagel, and S. Santosh. 2015. “Interpreting Factor Models.” Working paper, Ross School of Business and University of Maryland.
- Longstaff, F. A. 1995. Option Pricing and the Martingale Restriction. *Review of Financial Studies* 8: 1091–1124.
- Glasserman, P., C. Kang, and W. Kang. 2015. Stress Scenario Selection by Empirical Likelihood. *Quantitative Finance* 15: 25–41.
- Rubinstein, M. 1994. Implied Binomial Trees. *Journal of Finance* 49: 771–818.
- Schneider, P., and F. Trojani. 2015. Fear Trading. Technical Report, University of Lugano, and University of Geneva.
- Siriwardane, E. 2013. The Probability of Rare Disasters: Estimation and Implications. Technical Report, NYU Stern School of Business.
- Snow, K. 1991. Diagnosing Asset Pricing Models using the Distribution of Assets Returns. *Journal of Finance* 46: 955–883.
- Stutzer, M. 1996. A Simple Nonparametric Approach to Derivative Security Valuation. *Journal of Finance* 51: 1633–1652.
- Welch, I., and A. Goyal. 2008. A Comprehensive Look at the Empirical Performance of Equity Premium Prediction. *Review of Financial Studies* 21: 1455–1508.