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The option CAPM and the performance of hedge funds

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Abstract We evaluate the investment performance of hedge funds using an asset pricing model that is characterized by a piecewise-linear stochastic discount factor, and which we estimate using the generalized method of moments by minimizing the Hansen–Jagannathan distance. Our results show that, once non-linearities and public information are taken into account, there is only evidence of positive performance for the overall hedge fund index, equity-market neutral strategy and the global macro strategy.

Keywords Hedge funds · Non-linear return structure · Performance evaluation

JEL Classification C1 · C5 · G1

1 Introduction

The common wisdom about hedge funds is that they generate high returns with low volatility and low correlation with the market, and therefore they provide great diversification value if added to your portfolio (see e.g. Lhabitant 2004). However, episodes

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involving Long-Term Capital Management (LTCM) in 1998, Amaranth in 2006, and the Credit Crunch of 2008–2009 have raised questions about the true nature of their risks. Recent literature suggests that hedge fund returns exhibit non-linear structures and option-like features.[1] Fung and Hsieh (2001) analyze trend-following strategies and show that their payoffs are related to those of an investment in a lookback straddle. Mitchell and Pulvino (2001) show that returns to risk arbitrage are similar to those obtained from selling uncovered index put options. Agarwal and Naik (2004) extend these results and show that, in fact, a wide range of equity-oriented hedge fund strategies exhibit this non-linear payoff structure. Diez de los Rios and Garcia (2010) estimate the portfolio of options that best approximates the returns of a given hedge fund accounting for this search in the statistical testing of the non-linearity. Agarwal et al. (2009) show that hedge funds are substantially exposed to higher-moment risk.

When such non-linearities are present, it is important to take them into account when exploring whether the value of the cash flows generated by a hedge fund, net of all fees (management and performance-based incentive fees), is greater than the amount entrusted to the fund manager. Otherwise, we may interpret as a positive performance (alpha) what is in reality a non-linear risk exposure (beta).

To this end, we evaluate the investment performance of hedge funds using an asset pricing model that is characterized by a piecewise-linear stochastic discount factor. As shown in Vanden (2004), such a stochastic discount factor can be rationalized within an asset pricing model by imposing nonnegativity wealth constraints on a group of agents that have linear risk tolerance. Under such an approach, abnormal performance is measured by the expected product of a fund's return and the stochastic discount factor. Similarly, we follow Dahlquist and Söderlind (1999) and Farnsworth et al. (2002) to use conditional information variables to set the benchmark for what can be considered "superior" information by a manager.

We estimate this stochastic discount factor using the generalized method of moments by minimizing the so-called Hansen–Jagannathan distance: the minimum distance between the stochastic discount factor of an asset pricing model and the admissible set of stochastic discount factors. In other words, we obtain parameter estimates of our model by trying to be as close as possible to the admissible stochastic discount factor. As a byproduct, we also test whether the stochastic discount factor is non-linear. However, standard hypothesis tests to determine whether there is an option term in the stochastic discount factor is not applicable. When this coefficient is zero, the parameter corresponding to the option strike is not identified. Thus, the usual critical values of a Student t-test cannot be used to establish whether there exists a non-linear relationship between the stochastic discount factor and the benchmark returns. To overcome this important problem, we compute the critical values corresponding to a bootstrap distribution. As noted by Kosowski et al. (2006), such a bootstrap approach is also necessary for proper inference due to the short history of returns that characterizes hedge fund databases, and the propensity of hedge funds to exhibit non-normally distributed returns.

[1] See an excellent survey by Stulz (2007) and the recent references therein.

We apply this methodology to a set of Credit Suisse/Tremont indexes of hedge fund categories including the overall hedge fund industry index, convertible arbitrage, dedicated short bias, emerging markets, equity market neutral, event driven, fixed-income arbitrage, global macro, long-short equity, and managed futures. Our results show that, once non-linearities and public information are taken into account, there is only evidence of positive performance for the overall hedge fund index, the equity-market neutral strategy and the global macro strategy. We also find strong evidence in favour of the existence of non-linearities in the stochastic discount factor.

The rest of this paper is organized as follows. In Sect. 2 we explain the stochastic discount factor approach to performance evaluation. Section 3 describes the piecewise linear stochastic discount factor and its estimation. Section 4 describes the data. Results on the estimation of the stochastic discount factor and its properties are found in Sect. 5, while the results on the hedge fund performance evaluation are presented in Sect. 6. Section 7 concludes. Appendix A presents the econometric details of our bootstrap procedures, and Appendix B describes the Credit Suisse/Tremont categories of hedge funds.

2 Performance evaluation with stochastic discount factors

Our ultimate goal is to find whether a given fund provides a positive performance to investors, that is, whether the value of the cash flows generated by a fund, net of all fees, is greater than the amount entrusted to the fund manager. To do so, we follow the approach in Chen and Knez (1996), Dahlquist and Söderlind (1999) and Farnsworth et al. (2002) to estimate a stochastic discount factor that prices some basic ("benchmark") assets to then test whether it also prices the funds.

2.1 Model setup

Assume the existence of n basic assets to be priced with gross returns (that is, payoff divided by price) collected in vector $\mathbf{R}_{1,t+1}$. We will refer to them indistinctively as basic, benchmark or primitive-assets because investors can form portfolios of these certain assets. The hypothesis of absence of arbitrage implies the existence of a set of M stochastic discount factors[2] (SDFs) which price every one of these primitive-assets correctly through the following relationship:

$$E\left(m_{t+1}\mathbf{R}_{1,t+1}\middle|\,\mathbf{\Phi}_t\right) = \mathbf{1}_n \qquad \forall j, t > 0, \qquad \forall m_{t+1} \in M_{t+1}, \tag{1}$$

where m_{t+1} is the SDF at time $t+1$, M_{t+1} is the set of correct pricing kernels, $\mathbf{1}_n$ is an n-vector of ones, and $\mathbf{\Phi}_t$ denotes the information set available at time t.

Let $\mathbf{z}_t \in \Phi_t$ be a vector of predetermined instruments known at time t that help forecast either returns, the SDF, or both. We remain, for the moment, agnostic as to

[2] The SDF is alternatively known as pricing kernel or state price density and it is a concept related to the representative agent's nominal intertemporal marginal rate of substitution of consumption (See Cochrane 2005 for an extended discussion on the SDF).

the nature of these instruments. By the law of iterated expectations, one can replace $\mathbf{\Phi}_t$ by $\mathbf{z}_t$ in Eq. 1 to arrive at:

$$E\left(m_{t+1}\mathbf{R}_{1,t+1}\middle|\,\mathbf{z}_t\right) = \mathbf{1}_n \qquad \forall j, t > 0, \qquad \forall m_{t+1} \in M_{t+1}, \tag{2}$$

which implies that it should not be possible to find any sort of asset misspricing when using the information contained in the vector $\mathbf{z}_t$. In the extreme case where we let $\mathbf{z}_t$ be a constant, we obtain the unconditional version of the pricing Eq. 1:

$$E\left(m_{t+1}\mathbf{R}_{1,t+1}\right) = \mathbf{1}_n \qquad \forall j, t > 0, \qquad \forall m_{t+1} \in M_{t+1}. \tag{3}$$

As noted by Farnsworth et al. (2002), a proper choice of the conditioning variables in $\mathbf{z}_t$ allows the researcher to set the benchmark for what can be considered "superior" information by a manager. To illustrate this point, let's now assume that there is a fund whose manager invests in the primitive-assets on the basis of such a public information in $\mathbf{z}_t$. Thereby, we can express the return generated by the hedge fund whose performance we want to evaluate, $R_{2,t+1}$, as:

$$R_{2,t+1} = \mathbf{w}(\mathbf{z}_t)'\mathbf{R}_{1,t+1} \text{ such that } \mathbf{w}(\mathbf{z}_t)'\mathbf{1}_n = 1, \tag{4}$$

where the portfolio weight vector $\mathbf{w}(\mathbf{z}_t)$ is simply a function of the publicly available information at time t in the vector of instruments $\mathbf{z}_t$. Following Chen and Knez (1996), we now define the fund's conditional alpha, a traditional performance measure, as:

$$\alpha_t \equiv E\left(m_{t+1}R_{2,t+1}\middle|\,\mathbf{z}_t\right) - 1. \tag{5}$$

Substituting (4) into the above definition of the fund's alpha and rearranging we obtain:

$$\begin{aligned}\alpha_t &= E\left[m_{t+1}\mathbf{w}(\mathbf{z}_t)'\mathbf{R}_{1,t+1}\middle|\,\mathbf{z}_t\right] - 1 \\ &= \mathbf{w}(\mathbf{z}_t)'E\left[m_{t+1}\mathbf{R}_{1,t+1}\middle|\,\mathbf{z}_t\right] - 1 \\ &= \mathbf{w}(\mathbf{z}_t)'\mathbf{1}_n - 1 = 0,\end{aligned} \tag{6}$$

where the first equality in the last line comes from the fact the the SDF correctly prices all the benchmark assets. A fund displaying a positive alpha will then indicate the existence of a manager who possesses investment information or skills such that he can generate superior portfolio returns. Otherwise, we should find that the SDF also prices the fund and, therefore, we could stack Eqs. 2 and 6 to arrive to:

$$E\left[\begin{array}{c} m_{t+1}\mathbf{R}_{1,t+1} \\ m_{t+1}R_{2,t+1} \end{array}\middle|\,\mathbf{z}_t\right] = E\left(m_{t+1}\mathbf{R}_{t+1}\middle|\,\mathbf{z}_t\right) = \mathbf{1}_{n+1}, \tag{7}$$

where $\mathbf{R}_{t+1} = \left(\mathbf{R}'_{1,t+1}, R_{2,t+1}\right)'$ is a vector that stacks the returns on the original set of factors and the return on the hedge fund. We will use Eq. 7 with $\mathbf{z}_t$ being equal to a constant to estimate and test whether the fund provides a positive performance.[3]

Notice however that, since the true SDF is unknown, a fund's performance is model-dependent and different choices of the pricing kernel will lead to different performance implications for a given fund. In particular and as noted by Hansen and Jagannathan (1997), an asset pricing model can simply be understood as a SDF proxy, y_{t+1}, such that if the model is true then $y_{t+1} \in M_{t+1}$. If this is the case, not only Eq. 1 holds for all n benchmark assets once we replace m_{t+1} for y_{t+1}, but one can use this pricing proxy to calculate the fund's performance.

For the sake of tractability, we will focus on asset pricing models in which the SDF proxy can be written as an affine function of a $l \times 1$ vector of state variables, $\mathbf{f}_t$:

$$y_{t+1} = \lambda_0 + \mathbf{f}'_{t+1}\boldsymbol{\lambda}_1 = \mathbf{x}'_{t+1}\boldsymbol{\lambda}, \tag{8}$$

where λ_0 is a constant, $\boldsymbol{\lambda}_1$ is a $l \times 1$ vector of coefficients, $\boldsymbol{\lambda} = \left(\lambda_0, \boldsymbol{\lambda}'_1\right)'$ and $\mathbf{x}_{t+1} = \left(1, \mathbf{f}'_{t+1}\right)'$. A typical example of an asset pricing model with an affine pricing kernel is the Capital Asset Pricing Model (CAPM) where the state variable is the return on the portfolio of aggregate wealth (the market portfolio).

2.2 Estimation

When the asset pricing model is false and $y \notin M$, Hansen and Jagannathan (1997) note there is a strictly positive distance between the SDF proxy, y, and the set of true SDFs, M. This Hansen–Jagannathan (HJ) distance is defined as:

$$\delta = \min_{m \in M} \|m - y\|, \tag{9}$$

where $\|X\| = E\left(X^2\right)^{1/2}$ is the standard L^2 norm, and it can be interpreted as a measure of the maximum pricing error of a portfolio that has a unit second moment.

One can obtain $\widehat{\boldsymbol{\lambda}}$, an estimate of the vector of coefficients $\boldsymbol{\lambda}$ in Eq. 8 by minimizing this distance. In other words, we can estimate the parameters of the model by finding the closest pricing proxy to the true SDF. Such an approach has the advantage of being closely related to a generalized method of moments (GMM) estimation problem. To see this point, we can solve (9) as in Hansen and Jagannathan (1997) to arrive at:

$$\delta = \left[E\left(y_t\mathbf{R}_t - \mathbf{1}_n\right)' E\left(\mathbf{R}_t\mathbf{R}'_t\right)^{-1} E\left(y_t\mathbf{R}_t - \mathbf{1}_n\right)\right]^{1/2}. \tag{10}$$

[3] Additionally, we could use returns scaled by the corresponding instrument in $\mathbf{z}_t$. These returns could be interpreted as the returns to managed portfolios. In order to keep the number of benchmark assets to a minimum, we leave this point for further research.

Now define the vector of pricing errors of the benchmark assets as:

$$\mathbf{g}(\boldsymbol{\lambda}) = E\left[y_t \mathbf{R}_t - \mathbf{1}_n\right],$$

where its sample counterpart is:

$$\overline{\mathbf{g}}_T(\boldsymbol{\lambda}) = \frac{1}{T}\sum_{t=1}^{T} y_t \mathbf{R}_t - \mathbf{1}_n,$$

and let $\mathbf{U}_T$ to be a sample estimate of $\mathbf{U} = E\left(\mathbf{R}_t \mathbf{R}_t'\right)$. Then, by squaring Eq. 10, and replacing population objects by their corresponding sample counterparts, it is clear that one can obtain an estimate of $\boldsymbol{\lambda}$ as:

$$\widehat{\boldsymbol{\lambda}} = \arg\min_{\boldsymbol{\lambda}} \widehat{\delta}^2 = \overline{\mathbf{g}}_T(\boldsymbol{\lambda})' \mathbf{U}_T^{-1} \overline{\mathbf{g}}_T(\boldsymbol{\lambda}), \tag{11}$$

which is a standard GMM problem which uses $\frac{1}{T}\sum\left(\mathbf{R}_t \mathbf{R}_t'\right)^{-1}$ as the weighting matrix.

The main advantage of models where the SDF is affine in the parameters is that its estimation problem can be solved analytically. Specifically, solving for $\boldsymbol{\lambda}$ in the first order condition of the problem in (11) gives:

$$\widehat{\boldsymbol{\lambda}} = \left(\mathbf{D}_T' \mathbf{U}_T^{-1} \mathbf{D}_T\right)^{-1} \left(\mathbf{D}_T' \mathbf{U}_T^{-1} \mathbf{1}_n\right), \tag{12}$$

where $\mathbf{D}_T$ is the gradient of the sample pricing errors with respect to the parameter vector:

$$\mathbf{D}_T \equiv \frac{\partial \overline{\mathbf{g}}_T(\boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}'} = \frac{1}{T}\sum_{t=1}^{T} \mathbf{R}_t \mathbf{x}_t' \xrightarrow{a.s.} \mathbf{D} \equiv E\left(\mathbf{R}_t \mathbf{x}_t'\right).$$

Moreover, the asymptotic distribution of $\widehat{\boldsymbol{\lambda}}$ is given by:

$$\sqrt{T}\widehat{\boldsymbol{\lambda}} \xrightarrow{a.s.} N\left(\boldsymbol{\lambda}, V^{\boldsymbol{\lambda}}\right), \tag{13}$$

being $\mathbf{V}^{\boldsymbol{\lambda}} = \left(\mathbf{D}'\mathbf{U}^{-1}\mathbf{D}\right)^{-1} \mathbf{D}'\mathbf{U}^{-1}\mathbf{S}\mathbf{U}^{-1}\mathbf{D} \left(\mathbf{D}'\mathbf{U}^{-1}\mathbf{D}\right)^{-1}$, and:

$$\mathbf{S} = \sum_{j=-\infty}^{\infty} E\left[\mathbf{g}_t(\boldsymbol{\lambda}) \mathbf{g}_{t+j}(\boldsymbol{\lambda})'\right] \text{ with } \mathbf{g}_t(\boldsymbol{\lambda}) = y_t \mathbf{R}_t - \mathbf{1}_n.$$

Using these results, we could use a Wald statistic to test if any element in the vector of parameters $\boldsymbol{\lambda}$ is equal to zero, that is $H_0 : \lambda_j = 0$ against $H_1 : \lambda_j \neq 0$:

$$W_T^{\lambda_j} = T \times \frac{\widehat{\lambda}_j^2}{\mathbf{V}_{T,jj}^{\boldsymbol{\lambda}}}, \tag{14}$$

where $\mathbf{V}^{\boldsymbol{\lambda}}_{T,jj}$ is the (j, j)-th element of an estimate of matrix $\mathbf{V}^{\boldsymbol{\lambda}}$. This Wald statistic $W_T^{\lambda_j}$ will have an approximate chi-square distribution with one degree of freedom (the number of restrictions) in large samples, and can be used to test whether factor j is a determinant variable of the pricing kernel.

Similarly, the asymptotic distribution of the pricing errors $\mathbf{g}_T(\widehat{\boldsymbol{\lambda}})$ is given by:

$$\sqrt{T}\overline{\mathbf{g}}_T(\widehat{\boldsymbol{\lambda}}) \xrightarrow{a.s.} N\left(\mathbf{0}_{n+1}, \mathbf{V}^{\mathbf{g}}\right), \tag{15}$$

where $\mathbf{V}^{\mathbf{g}} = \left[\mathbf{I}_{n+1} - \mathbf{D}\left(\mathbf{D}'\mathbf{U}^{-1}\mathbf{D}\right)^{-1}\mathbf{D}'\mathbf{U}^{-1}\right]\mathbf{S}\left[\mathbf{I}_{n+1} - \mathbf{D}\left(\mathbf{D}'\mathbf{U}^{-1}\mathbf{D}\right)^{-1}\mathbf{D}'\mathbf{U}^{-1}\right]'$. Thus we can construct another Wald test for the null hypothesis that the pricing errors are all equal to zero, that is, $H_0 : \mathbf{g}(\widehat{\boldsymbol{\lambda}}) = 0$ against $H_1 : \mathbf{g}(\widehat{\boldsymbol{\lambda}}) \neq 0$:

$$W_T^{\mathbf{g}} = T \times \overline{\mathbf{g}}'_T(\widehat{\boldsymbol{\lambda}})\left(\mathbf{V}_T^{\mathbf{g}}\right)^{-1}\overline{\mathbf{g}}_T(\widehat{\boldsymbol{\lambda}}), \tag{16}$$

being $\mathbf{V}_T^{\mathbf{g}}$ a sample estimate of the asymptotic variance of the pricing errors, $\mathbf{V}^{\mathbf{g}}$. Notice however that $\mathbf{V}_T^{\mathbf{g}}$ only has rank $n-l$, so we have to use its pseudo inverse when computing this statistic.[4] This Wald statistic $W_T^{\mathbf{g}}$ will have an approximate chi-square distribution with $n-l$ degrees of freedom (the number of restrictions) in large samples. While this approach does not provide a direct test of whether the HJ-distance is equal to zero, it is still important to notice that if $\mathbf{g}(\boldsymbol{\lambda}) = \mathbf{0}_{n+1}$, then the HJ-distance is equal to zero by construction. A direct test of such a null hypothesis, $H_0 : \delta = 0$, would however be more complicated given that the distribution of δ is not standard under the assumption that the true δ equals zero. Specifically, Jagannathan and Wang (1996) demonstrate that the distribution of $T\delta^2$ involves a weighted sum of $n - l\,\chi^2(1)$ statistics. The computation of critical values for such a test requires the use of simulation methods.

Finally, notice that the last element of $\overline{\mathbf{g}}_T$ corresponds to an estimate of the fund's unconditional alpha:

$$\mathbf{e}'_{n+1}\overline{\mathbf{g}}_T(\widehat{\boldsymbol{\lambda}}) = \frac{1}{T}\sum_{t=1}^{T}\left[y_{t+1}(\widehat{\boldsymbol{\lambda}})R_{2,t+1} - 1\right] = \alpha(\widehat{\boldsymbol{\lambda}}) \equiv \widehat{\alpha}, \tag{17}$$

where $\mathbf{e}_{n+1}$ is a vector of zeros with a one in the $(n+1)$th position. Using the results in (15), we can construct a t statistic to test the null hypothesis that the performance of the fund is zero, $H_0 : \widehat{\alpha} = 0$, against the alternative that the fund provides a positive performance $H_1 : \widehat{\alpha} > 0$:

$$t_T^{\alpha} = \frac{\mathbf{e}'_{n+1}\overline{\mathbf{g}}_T(\widehat{\boldsymbol{\lambda}})}{\sqrt{\mathbf{e}'_{n+1}\mathbf{V}_T^{\mathbf{g}}\mathbf{e}_{n+1}/T}}, \tag{18}$$

where this test statistic will have a $N(0, 1)$ distribution in large samples.

[4] We follow Dahlquist and Söderlind (1999) to impose the known rank of $\mathbf{V}_T^{g}$ when computing its pseudo-inverse.

2.3 Conditional models

So far we have focused on unconditional implications of our asset pricing model because the parameters of the pricing proxy in Eq. 8, $\boldsymbol{\lambda}$, are constant. Such an approach when estimating a SDF suffers from two criticisms as noted by Hodrick and Zhang (2001). One is that we are only focusing on unconditional risk premiums. The second one is that, by leaving the parameters constant, the model force the prices of risks to be constant across business cycles. In order to solve these issues, it is standard in the literature to assume that the parameters in the SDF are linear functions of a set of conditioning variables, typically, macroeconomic variables (see Ferson and Harvey 1999; Hodrick and Zhang 2001, among others). For example, we can allow them to vary with some element z_t in the information set at time t:

$$\begin{aligned} y_{t+1} &= \mathbf{x}'_{t+1}\boldsymbol{\lambda}(z_t) \\ &= (\lambda_{01} + \lambda_{02}z_t) + \mathbf{f}'_{t+1}(\boldsymbol{\lambda}_{11} + \boldsymbol{\lambda}_{12}z_t) \\ &= \lambda_{01} + \lambda_{02}z_t + \mathbf{f}'_{t+1}\boldsymbol{\lambda}_{11} + (\mathbf{f}_{t+1}z_t)'\boldsymbol{\lambda}_{12}, \end{aligned} \tag{19}$$

where we follow Hodrick and Zhang (2001) to focus on one conditioning variable at a time in order to keep the estimation tractable.[5]

As noted by Cochrane (2005), scaling the price of factors is equivalent to scaling the factors. Thus, we can express conditional models as unconditional models with constant parameters in the following way:

$$y_{t+1} = \widetilde{\mathbf{x}}'_{t+1}\widetilde{\boldsymbol{\lambda}},$$

where now we have the augmented set of factors given in $\widetilde{\mathbf{x}}_{t+1} = \left[1, z_t, \mathbf{f}'_{t+1}, (\mathbf{f}_{t+1}z_t)'\right]'$. Hence, (19) can be easily estimated using the results above.

3 A piecewise linear stochastic discount factor

As noted above, the main advantage of the models presented in the previous section is that its estimation problem can be solved analytically. Yet, the literature has identified several problems with these linear asset-pricing models when used for the task of performance evaluation. First, these models restrict the relationship between risk factors and returns to be linear, and thus do not properly evaluate assets with non-linear payoffs. Therefore, we may interpret as a positive performance (alpha) what is in reality a non-linear risk exposure (beta). Second, performance measures based on linear models, such as Jensen's alpha and Sharpe ratios, can be manipulated by taking positions in the derivatives market (see, for example, Jagannathan and Korajczyk 1986; Goetzmann et al. 2007). Again, the potential positive value exhibited by a fund can

[5] Notice that the number of parameters to be estimated grows geometrically with the number of conditioning variables. Given that our Hedge Fund database is characterized by a relatively short span of data, including several conditioning variables at the same time could negatively affect the finite-sample properties of our estimators.

simply be the fair price to be paid for these options. These two problems are especially relevant in our setup, because several studies have put forward the non-linear structure and option-like features of returns associated with hedge fund strategies and, second, these usually take positions in derivative securities.

To solve such problems while remaining in the tractability realm, several authors have proposed pricing proxies that are polynomial functions on the return of the market portfolio. For example, Harvey and Siddique (2000) propose a quadratic SDF while Dittmar (2002) approximates the true pricing kernel using a third-order Taylor expansion. Such models can still be thought as being linear in the augmented set of factors given by the return on the portfolio of aggregate wealth, the square, and, if needed, its cubic power.

We, on the other hand, focus on the case of the option CAPM where the SDF simply takes the form of a piecewise linear function on the return of the market portfolio $R_{M,t}$ as in Vanden (2004):

$$y_{t+1} = \lambda_0 + \lambda_1 R_{M,t+1} + \lambda_2 \max(R_{M,t+1} - k, 0), \tag{20}$$

which can be expressed in vector notation as:

$$y_{t+1} = \lambda_0 + \mathbf{f}_{t+1}(k)'\boldsymbol{\lambda}_1 = \mathbf{x}_{t+1}(k)'\boldsymbol{\lambda}, \tag{21}$$

where $\mathbf{x}_t(k) = \left[1, \mathbf{f}_t(k)'\right]' = \left(1, R_{M,t}, R_{M,t} - k\right)'$ and $\boldsymbol{\lambda} = \left[1, \boldsymbol{\lambda}_1'\right]' = (\lambda_0, \lambda_1, \lambda_2)'$. In particular, notice that the last term in (20), $\max(R_{m,t+1} - k, 0)$, is the payoff at expiration on an index call option with exercise price k when the current value of the stock market index is one. We will also refer to the strike parameter, k, as the "moneyness of the option." As shown in Vanden (2004), such a pricing kernel can be rationalized within an asset pricing model by imposing nonnegativity wealth constraints on a group of agents that have linear risk tolerance.

When the strike of option, k, is known a priori we can use Eq. 12 to obtain an estimate of the parameters of the SDF as

$$\widehat{\boldsymbol{\lambda}}(k) = \left[\mathbf{D}_T(k)'\mathbf{U}_T^{-1}\mathbf{D}_t(k)\right]^{-1}\left[\mathbf{D}_T(k)'\mathbf{U}_T^{-1}\mathbf{1}_n\right], \tag{22}$$

where now the gradient of the sample pricing errors with respect to the parameter vector is given by:

$$\mathbf{D}_T(k) = \frac{1}{T}\sum_{t=1}^{T}\mathbf{R}_t\mathbf{x}_t'(k).$$

The dependence of $\widehat{\boldsymbol{\lambda}}(k)$ on the moneyness of the option indicates that there is a different estimate of the parameters of the SDF for each different choice of k.

Similarly, when k is known we can use a Wald statistic such as the one in Eq. 14 to test if the SDF is linear $H_0 : \lambda_2 = 0$ against $H_1 : \lambda_2 \neq 0$:

$$W_T^{\lambda_2}(k) = T \times \frac{\left[\widehat{\lambda}_2(k)\right]^2}{\mathbf{V}^{\boldsymbol{\lambda}}_{T,22}(k)} \tag{23}$$

where $\mathbf{V}^{\boldsymbol{\lambda}}_T(k)$ is a sample estimate of the asymptotic variance-covariance matrix of the parameters in the SDF which, again, depend on the a priori choice of k. This Wald statistic $W_T^{\lambda_2}(k)$ will have an approximate chi-square distribution with one degree of freedom (the number of restrictions) in large samples.

The strike of the option, k, is unfortunately not known and has to be estimated along with the rest of parameters of the model. In practice, an estimate of k can be found sequentially through concentration. For a given value of the strike of the option k, we minimize the HJ-distance using GMM as if it were known. We then search over the possible values of k for the one that minimizes the HJ-distance to get our estimate of this parameter. [6] Note however that if we were to test the non-linearity of the SDF in Eq. 20, that is, we were to test $H_0 : \lambda_2 = 0$, the corresponding test statistic would have a non-standard distribution because the strike of the option, k , would not be identified. In this case, we could we follow Davies (1977, 1987) and compute the Wald test statistic, $W_T^{\lambda_2}(k)$, for each possible value of k and then focus on the supremum value of such a sequence, $W_T^{\lambda_2} = \sup_k W_T(k)$, as done in Diez de los Rios and Garcia (2010). This statistic is known as "supWald" and it has an asymptotic distribution that is non-standard. In particular, using empirical process theory, Hansen (1996) derives the asymptotic distribution of this test under the null hypothesis of linearity and provides a simulation method to compute the various distributions. In addition, the standard distribution of the Wald test statistic for the null hypothesis that the pricing errors are zero in Eq. 16, and that of the t-statistic for the null hypothesis that the fund has no positive performance in Eq. 18 will remain invalid because k is chosen in a data-dependent procedure. In these two cases, we will have to compute a "supWald" and a "sup-t " statistic.

Since a fair amount of computation is involved in calculating the asymptotic p-values of our main hypothesis of interest using the simulation methods, in this paper we have decided instead to directly calculate the p-values using the bootstrap since, for almost the same amount of computational work, this could provide a better approximation to the finite sample distributions of the statistics than the first-order asymptotics behind the results in Hansen (1996).[7] As noted by Kosowski et al. (2006), there are many reasons why the bootstrap is necessary for proper inference such as the propensity of returns to be non-normally distributed (especially those representing non-linear payoffs such as those of the option portfolios or hedge fund indexes) or,

[6] In particular, we restrict our search to the values of k lying between the $\tau = 0.96$ and $\tau = 1.04$ with steps of 0.001. That is, we are searching options from 4% in-the-money to 4% out-of-the-money.

[7] A caveat to bear in mind is that certain technical conditions (such as the existence of an Edgeworth expansion) are needed for the bootstrap distribution of an asymptotically pivotal statistic to achieve a higher rate of convergence to the sampling distribution than the first-order asymptotic approximation. These conditions have not been verified for our setup (at least up to our knowledge) so it is not clear whether the boostrap will achieve an accelerated rate of convergence. Yet Diebold and Chen (1996), in an analogous inference problem to the one presented here such as the study of the Andrews structural change test, find that the bootstrap yields an improvement over the asymptotic distribution.

in our case, the short history of returns that characterizes hedge fund data. Specific details on our bootstrap computations can be found in Appendix A.

4 Data

In this section we describe the different datasets used in our study: primitive assets, hedge fund returns, conditioning information and risk factors. Our sample starts in January 1994 and ends in June 2008 for a total of 174 monthly observations.

4.1 Benchmark assets

In our set of test assets, we include the six Fama–French portfolios sorted by size and book-to-market. We proxy for the risk-free security by including the gross return on the 30-day Treasury bill.[8] In this way, we are able to pin down the mean of the SDF to a reasonable value.

In addition, we consider another two sets of non-linear test assets. First, we include the monthly returns on the four (Agarwal and Naik 2004) option portfolios. These consists of highly liquid at-the-money (ATM) and out-of-the-money (OTM) European call and put options on the S&P 500 composite index trading on the Chicago Mercantile Exchange.[9] For example, Agarwal and Naik (2004) construct the time series of returns of their ATM call portfolio as follows. On the first trading day in January, buy an ATM call option that expires in February. On the first trading day in February, sell the option bought a month ago and buy another ATM call option that expires on March, and repeat this trading pattern every month.

Second, we also consider the returns on the Fung and Hsieh (2001) so-called "Primitive Trend-Following Strategy" (PTFS) factors on five different markets: bond, currency, commodity, short-term interest rate and stock index.[10] These are basically returns to investing on a lookback straddle on the corresponding asset mentioned above.[11] The specific computational details of these returns can be found in Fung and Hsieh (2001).

4.2 Risk factors and conditioning variables

The main risk factor in the option CAPM SDF specification in Eq. 20 is the (gross) return on the portfolio of aggregate wealth which we proxy by the CRSP value-weighted NYSE, AMEX, and NASDAQ combined index.

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[8] The data for the returns on these assets and the risk factors described in the next subsection are obtained from Ken French's website.

[9] We thank Vikas Agarwal for kindly providing us with this dataset.

[10] The data for the returns on these assets are obtained from David Hsieh's website.

[11] A straddle is an option-based strategy whereby an investor holds a position in both a call and put with the same strike price and expiration date. A lookback call (put) option is an option whose payoff is dependent on the maximum (minimum) of the asset price achieved over the life of the option.

Regarding conditioning variables, we use two instruments to capture the variability of the prices of risk over the business cycle. We first include a short-term interest rate (again proxied by the return on the 30-day Treasury bill) as this variable is considered to have predicting power on security returns over time and, therefore, it has been previously used in studies of managed portfolios performance (see e.g. Farnsworth et al. 2002). To allow the prices of risk to change over the business cycle, we use a second instrument that consists of a dummy variable that takes the value one when the corresponding month has been declared as recession by the National Bureau of Economic Research (NBER).

4.3 Hedge fund returns

In this paper, we focus on the evaluation of nine different hedge fund strategies, and, to this end, we use monthly returns on the Credit-Suisse/Tremont indexes for the overall hedge fund industry and the following categories: (1) convertible arbitrage; (2) dedicated short bias; (3) emerging markets; (4) equity market neutral; (5) event driven; (6) fixed-income arbitrage; (7) global macro; (8) long-short equity; (9) managed futures.[12] Detailed descriptions of investment strategies followed by hedge funds in these categories can be found in the Appendix B. The Credit-Suisse/Tremont indexes are value-weighted (by the assets under management) and, therefore, give more weight to large funds.[13]

However, such a dataset has the main drawback of not correcting for two well-known biases associated with hedge fund data. The first one is a backfilling or instant-history bias, whereby the database backfills the historical return data of a fund before its entry into the database. The hypothesis is that a manager will report to the database vendor only after obtaining a good track record of returns over the first periods of the hedge fund life. The second one is the so-called liquidation bias. Many funds disappear from the database during the sample period for various reasons that may not have the same consequences in terms of monetary loss for the investor. An issue of concern is when an underperforming fund ceases to report in order to hide bad results and avoid a massive withdrawal from investors. Recently, Agarwal et al. (2010) have brought some new evidence about these self-reporting biases, both at initiation and termination dates. They found that in average self-reporting and non-reporting funds do not differ significantly in return performance but that reporting funds incur a substantial deterioration in performance, after both starting and termination dates. Diez de los Rios and Garcia (2010) correct for the backfilling and liquidation biases based on some conservative

[12] The data for the returns on these hedge fund strategies are obtained from www.hedgeindex.com.

[13] The Credit Suisse/Tremont hedge fund indexes that we use in this paper are constructed with the objective of being an accurate barometer of the hedge fund industry, and thus they also use data on funds that are closed to new investment. While Credit Suisse/Tremont also provides data on an investable index that only includes open funds, it only starts in August 2003 which would greatly reduce the number of observations if we were to apply our methodology to such dataset. In any case, we find that both indexes present a very similar behavior over the period they overlap, so we expect our conclusions to also extend to the case of the investable index.

assumptions.[14] The estimated average bias for equally-weighted indices built from the individual funds in the TASS database from January 1966 to March 2004 vary from 2% for convertible arbitrage to a bit more than 4% for long-short equity hedge and emerging markets.[15] They also found that the biases are in general more severe for value-weighted indices built from the same database. We also expect that the reporting bias should be greater for hedge fund indices since only a small number of funds report to the index providers. When analyzing our empirical results we should therefore keep in mind that our performance estimates may suffer from these potential biases.

4.4 Summary statistics

Table 1 reports summary statistics of our data. Panel A presents the distributional characteristics of the primitive assets. Average monthly net returns on the Fama–French portfolios range from 0.64 to 1.26% (7.68–15.12% per year) and present the lowest volatility among the three groups of benchmark assets. On the other hand, skewness tend to be negative indicating that the mass of the return distribution is concentrated on the right of the mean. Similarly, there is evidence of excess kurtosis in Fama–French returns, meaning that the peakedness is higher and the tails are thicker than in normal distributions. As for the straddle-like PTFS factors in Fung and Hsieh (2001), mean returns tend to be negative except in the case of the currency (which is close to zero) and the short-term interest rate straddle (which presents a monthly mean return of 1.41%, a 16.92% per year). As a difference with the Fama–French portfolios, these straddle-like returns are around three times more volatile, present positive skewness, and the tails of their return distribution are even thicker (higher excess kurtosis). Finally, all four Agarwal–Naik option portfolio strategies present negative mean returns and their standard deviations rank the highest among the set of primitive assets. The skewness coefficients are all positive while only the OTM option returns present positive excess kurtosis. The excess kurtosis coefficient for the ATM option returns is negative albeit small.

Panel B reports summary statistics of the factors and conditioning instruments. The market portfolio return presents distributional characteristics that are similar to those of the Fama–French portfolios. On the other hand, the low average return of the T-Bill rate (0.32%, that is, about 10% per year) is associated with a low monthly volatility of 0.14%.

Last, Panel C presents summary statistics of the hedge fund strategy returns. Average monthly net returns on the hedge fund strategies range from 0.03% (dedicated short bias) to 1.14% (global macro), that is from 0.36 to 13.68% per year. These numbers are consistent with those found for the Fama–French portfolios, yet the volatility of hedge fund returns tends to be lower. In terms of skewness, the results are less

[14] For the backfilling bias, they eliminate all data that precede the fund entry date in the database, as in Fung and Hsieh (2001). For the liquidation bias, they apply an extra zero return when the indicated reasons for not reporting are fund liquidation, fund not reporting to TASS, managers not answering requests, and other. In all other cases, particularly mergers and dormant funds, they do not apply any loss.

[15] The raw figures without bias correction are reported in the working paper version of the paper (see Working Paper 2006-31 from the Bank of Canada).

Table 1 Summary statistics

	Mean	SD	Skew.	Kurt.	Min.	Max.
A. Primitive assets						
Fama–French portfolios returns						
SL	0.64	7.18	0.00	1.51	−24.39	28.95
S2	1.19	4.75	−0.58	1.93	−18.90	15.46
SH	1.26	4.61	−0.72	1.92	−17.21	11.78
BL	0.84	4.40	−0.54	0.53	−14.38	9.60
B2	0.95	4.09	−0.68	2.12	−17.74	10.66
BH	0.90	4.07	−0.49	0.90	−13.08	12.34
Primitive trend following strategy returns						
Bond	−1.28	14.58	1.51	3.51	−25.36	68.86
Currency	0.01	18.61	1.30	2.95	−30.13	90.27
Commodity	−0.31	13.36	1.35	3.37	−23.04	64.75
Short-term interest rate	1.41	25.08	4.81	35.97	−30.60	221.92
Stock	−4.98	12.77	1.01	2.18	−30.19	46.15
Option portfolio returns						
ATM call	−2.32	77.90	0.61	−0.61	−99.18	216.91
ATM put	−4.57	83.85	0.78	−0.37	−99.12	246.85
OTM call	−19.41	86.90	1.75	3.27	−95.76	386.02
OTM put	−22.61	88.22	1.92	4.11	−96.46	400.33
B. Factors and conditioning information						
CRSP market portfolio	0.84	4.20	−0.74	0.90	−15.77	8.39
CRSP T-Bill	0.32	0.14	−0.53	−1.07	0.06	0.56
NBER recession dummy	0.09	0.29	2.85	6.19	0.00	1.00
C. CS/Tremont hedge fund returns						
Hedge fund industry	0.87	2.15	0.10	2.54	−7.55	8.53
1- Convertible arbitrage	0.65	1.41	−1.55	4.28	−5.93	3.57
2- Dedicated short bias	0.03	4.88	0.79	1.86	−8.69	22.71
3- Emerging markets	0.85	4.44	−0.71	5.30	−23.03	16.42
4- Equity market neutral	0.79	0.80	0.35	0.58	−1.15	3.26
5- Event driven	0.91	1.60	−3.12	21.93	−11.77	3.68
6- Fixed income arbitrage	0.47	1.17	−3.15	15.67	−6.96	2.07
7- Global macro	1.14	2.99	0.01	3.39	−11.55	10.60
8- Long/short equity hedge	0.97	2.83	0.19	4.02	−11.43	13.01
9- Managed futures	0.64	3.44	0.00	0.17	−9.35	9.95

This table shows the means, standard deviations (SD), skewness (Skew.), excess kurtosis (Kurt.), and minimum (Min.) and maximum (Max.) of monthly data on (A) the primitive assets returns, (B) the factors and conditioning information, and (C) the Credit Suisse/Tremont Hedge Fund Indexes over the period January 1994 to June 2008 (174 observations) uniform: the asymmetry coefficient tends to be positive albeit small except in the cases of convertible arbitrage, emerging markets, event driven and fixed-income arbitrage strategies. Hedge fund returns display large excess kurtosis coefficients.

5 Estimating the SDF models

Before exploring whether hedge funds provide positive performance to investors, we first report in this section summary statistics for fitted SDFs and pricing errors for both unconditional and conditional models computed from data on the benchmark assets only.

5.1 Summary statistics for fitted SDFs

In Table 2, we evaluate the fit of the different SDF models using only the sample of benchmark assets. As noted earlier in the paper, these models are estimated using monthly data for the period January 1994 to June 2008 for a total of 174 observations. The estimates of the HJ-distance are labeled "HJ-dist(δ)". The p-values of the test for the null hypothesis that the HJ-distance is equal to zero, $H_0 : \delta = 0$, are labeled "$p(\delta = 0)$ ". The p-values of the test for the null hypothesis that all the pricing errors are equal to zero, $H_0 : \mathbf{g} = \mathbf{0}$, are labeled "$p(\mathbf{g} = \mathbf{0})$". When estimating the piece-wise linear SDF in Eq. 20, we report the estimated moneyness of the option under the column labeled as "$\widehat{k}$". Accordingly, the p-values of the test for the null hypothesis that the true SDF is linear are labeled as "$p(y_t = \text{linear})$ ". All the p-values reported in these tables are computed using bootstrap distributions of the corresponding test statistics. Finally, we report summary statistics for the time series of the fitted SDFs.

Panel A of Table 2 summarize the results for the unconditional models. The first column of Panel A indicates that as we include an option term into the SDF specification and allow the moneyness of such an option to be freely estimated, the HJ-distance drops (0.712, 0.538, and 0.509 respectively). Yet in the best case, the option CAPM with an optimally estimated strike, the p-values of the test for the hypothesis that the HJ-distance is equal to zero is well below the cut-off value of 5. Such a result is confirmed when we test whether the pricing errors on all the primitive assets are jointly zero using the corresponding Wald test. Notice also that the p-values of the test for the hypothesis that the true SDF is linear are close to zero for both the case that the strike of the option in Eq. 20 is fixed to one and the case where it is optimally estimated. The optimally estimated strike is close to 0.98 which indicates that the option term in the SDF is 2% in-the-money. As for the summary statistics for the time series of the fitted SDFs, it is worth pointing out that the means of the three estimated SDF proxies are equal to the inverse of the mean of the gross T-Bill rate, indicating that including a proxy of the risk-free rate in the set of primitive assets effectively helps to pin down the mean of the SDF. Also, notice that as we increase the complexity of the model—first allowing for an option term in the SDF, and then optimally choosing for the strike of such an option– the standard deviation of the fitted SDF increases. However, neither of these three unconditional models present an estimated SDF that takes on negative values.

Table 2 Summary of stochastic discount factor models

	HJ-dist(δ)	$p(\delta = 0)$	$p(\mathbf{g} = \mathbf{0})$	$\widehat{k}$	$p(y_t = \text{linear})$	$E(y_t)$	SD(y_t)	Min(y_t)	Max(y_t)	$\#(y_t < 0)$
A. Unconditional models										
CAPM	0.712	0.000	0.000			0.997	0.118	0.784	1.466	0
Option CAPM										
$k = 1$	0.538	0.001	0.027		0.001	0.997	0.597	0.188	4.267	0
optimal k	0.509	0.002	0.017	0.981	0.001	0.997	0.653	0.207	5.343	0
B. Conditional models with scaled factors by short-term interest rate										
CAPM	0.608	0.032	0.396			0.995	1.425	−2.456	3.386	43
Option CAPM										
$k = 1$	0.491	0.125	0.213		0.001	0.996	0.986	−1.777	4.883	28
optimal k	0.383	0.344	0.236	0.960	0.087	0.996	1.351	−3.185	9.545	26
C. Conditional models with scaled factors by NBER recession dummy										
CAPM	0.701	0.001	0.001			0.997	0.378	−1.431	2.737	5
Option CAPM										
$k = 1$	0.511	0.007	0.030		0.000	0.997	0.744	−2.023	4.131	8
optimal k	0.463	0.015	0.030	0.973	0.000	0.997	0.949	−3.718	4.900	8

This table shows summary statistics for fitted SDFs. The estimates of the HJ-distance are labeled HJ-dist(δ). The p-values of the test for the null hypothesis that the HJ-distance is equal to zero, $H_0 : \delta = 0$, are labeled $p(\delta = 0)$. The p-values of the test for the null hypothesis that all the pricing errors are equal to zero, $H_0 : \mathbf{g} = 0$, are labeled $p(\mathbf{g} = \mathbf{0})$. When estimating the piecewise linear SDF in equation (20), the estimated moneyness of the option is presented under the column labeled as "$\widehat{k}$". Accordingly, the p-values of the test for the null hypothesis that the true SDF is linear are labeled as $p(y_t = \text{linear})$. $E(y_t)$ is the mean of the estimated SDF. Similarly, SD(y_t), Min(y_t), and Max(y_t), are the standard deviation, minimum and maximum of the estimated SDF. $\#(y_t < 0)$ is the number of times the estimated SDF becomes negative. The sample period is January 1994 to June 2008 (174 observations)

Results for the conditional models are reported in Panels B and C of Table 2. Panel B of Table 2 shows that by scaling factors by the short-term interest rate, the HJ-distance of the resulting model is smaller, vis-à -vis, the corresponding unconditional model. For example, the HJ-distance of the conditional CAPM model is reduced from a previous value of 0.712–0.608. While such a distance is statistically different from zero at the 5% level, evidence against this model is mixed because, using the corresponding Wald test, it is not possible to reject that all pricing errors are equal to zero. The HJ-distance of the conditional option CAPM with fixed strike equal to one is smaller than that of its unconditional counterpart, but it is still high and around 0.49. Although this is large number, it cannot be rejected that such a distance is equal to zero or that pricing errors are all zero. A similar pattern is found when we allow the strike of the option term in the SDF to be optimally estimated. In this case, the HJ-distance is the lowest across all models, 0.38, and again it cannot be rejected that it is equal to zero or that the pricing errors are all zero. The linearity of the SDF is rejected at the 1% level when the strike of the option is fixed, and rejected at the 10% level when this is optimally estimated. We attribute this increase in the p-value for the null hypothesis that the model is linear to the fact that the supWald test tend to lack power when the nuisance parameter, the strike of the option, tends to be close to its boundary (as it is the case of the estimated k).

The downside of these conditional models is that when scaling factors by the short-term rate, the estimated SDF turns negative a significant number of times. Such a result can also be found in Dahlquist and Söderlind (1999) and Farnsworth et al. (2002) whose estimated SDFs have more negative values when more factors are used.[16] This means that, the estimated SDF proxy assigns positive prices to negative payoffs and, therefore, it does not rule out arbitrage opportunities. For example, the CAPM SDF proxy becomes negative 43 times (almost a 25% of the total number of observations), while this number is reduced to 28 and 26 (around a 15% of total observations) when focusing on the non-linear models. Notice however that, as a difference to the unconditional models, the SDF becomes less volatile when we include a non-linear term into its specification but the HJ-distance does not increase accordingly. Such a reduction in the volatility of the estimated SDF is the explanation behind the fall in the number of times the SDF turns negative for the non-linear asset pricing models.

The results of the conditional models with scaled factors by the NBER recession dummy can be found in Panel C of Table 2. As in the case of the short-term rate instruments, the HJ-distance drops when scaling factors by the NBER dummy. However, such a reduction is not as strong as in the case of models scaled by the short-term interest rate. The HJ-distance estimate is 0.701 for the CAPM proxy, and 0.511 and 0.463 for the option CAPM proxies with fixed strike and estimated strike, respectively. The null hypothesis that the HJ-distance is equal to zero is rejected for each model.

[16] Dahlquist and Söderlind (1999) obtain estimates of the unconditional SDF subject to the non-negativity constraint by minimizing the HJ-distance using numerical methods. However, they did not impose positivity when estimating the conditional version of the SDF because of numerical problems when attempting to do so. In such a case, they show that a test of zero alpha without positivity constraint on the SDF can still be understood in terms of traditional mean-variance spanning tests, that is, tests of whether the mean-variance frontier of $\mathbf{R}_{1,t}$ intersects the mean-variance frontier of $\mathbf{R}_{1,t}$, and $\mathbf{R}_{2,t}$ (see Dahlquist and Söderlind 1999, p. 353).

The hypothesis that pricing errors are all equal to zero is equally rejected. Again, the estimated SDF turns negative when scaling the factors by the NBER dummy but the number of times this occurs is reduced to 5 in the case of the CAPM and to 8 in the case of the option CAPM (around a 5% of total observations).

5.2 Pricing errors for unconditional models

In order to provide additional information on the goodness-of-fit of the models, we also report pricing errors, **g**, for each one of the models considered in this paper. In particular, Fig. 1 reports the errors for the unconditional models, while Figs. 2 and 3 report the errors for the conditional models where the factors are scaled by the short-term interest rate and NBER recession dummy respectively. All figures are organized in the same way. Panel A reports the errors for the CAPM, while Panel B and C report the errors for the option CAPM when the strike is set to one, and optimally estimated respectively.

Panel A of Fig. 1 shows that the CAPM has problems in correctly pricing non-linear payoffs. While model errors for the six Fama–French portfolios range (in absolute terms) from 0% for the BL portfolio to 0.52% per month for portfolio SH,[17] the absolute pricing errors for the Agarwal–Naik option portfolios are all larger than 10%. In particular, the model tends to over-estimate the returns on these option portfolios. Similar problems are found when pricing the Fung–Hsieh PTFS portfolios. For example, the absolute model error for the lookback straddle strategy on the stock index is 5.02%, while those related to short-term interest rate and bond straddles are 1.56 and 1.34%, respectively. When comparing with the Fama–French portfolios, these are large pricing errors. The T-Bill rate is the only asset that is well priced.[18]

Including an option term in the SDF specification greatly reduces the pricing errors for the non-linear payoffs as shown in Panel B of Fig. 1 (notice the change of scale with respect to panel A). In addition, such an improvement in the pricing of non-linear assets does not come at the expense of the pricing of the Fama–French portfolios: model errors for these six portfolios now range (in absolute terms) from 0.06% for the BL portfolio to 0.42% per month for portfolio SH. As for the errors of the Agarwal–Naik portfolios, they are still large when compared to the errors of the Fama–French portfolios, but they are substantially lower than those obtained from the CAPM estimates. Now the option CAPM with fixed strike tend to underestimate the return on these portfolios, and their pricing errors range from 1.36% for the OTM call portfolio to 3.70% per month for the ATM put portfolio. As for the Fung–Hsieh PTFS portfolios, the pricing errors are similar to those found for the CAPM except for the error on the lookback straddle strategy on the stock index, whose absolute pricing error gets reduced to 2.28% per month from a previous level of 5.02%, and the error on the

17 These numbers are qualitatively similar to those found by Hodrick and Zhang (2001) when estimating the CAPM SDF using the 25 Fama–French portfolios.

18 Since the Treasury Bill return is much less volatile than the returns on the risky assets, the GMM approach tend to choose parameters of the SDF to price the risk-free asset well.

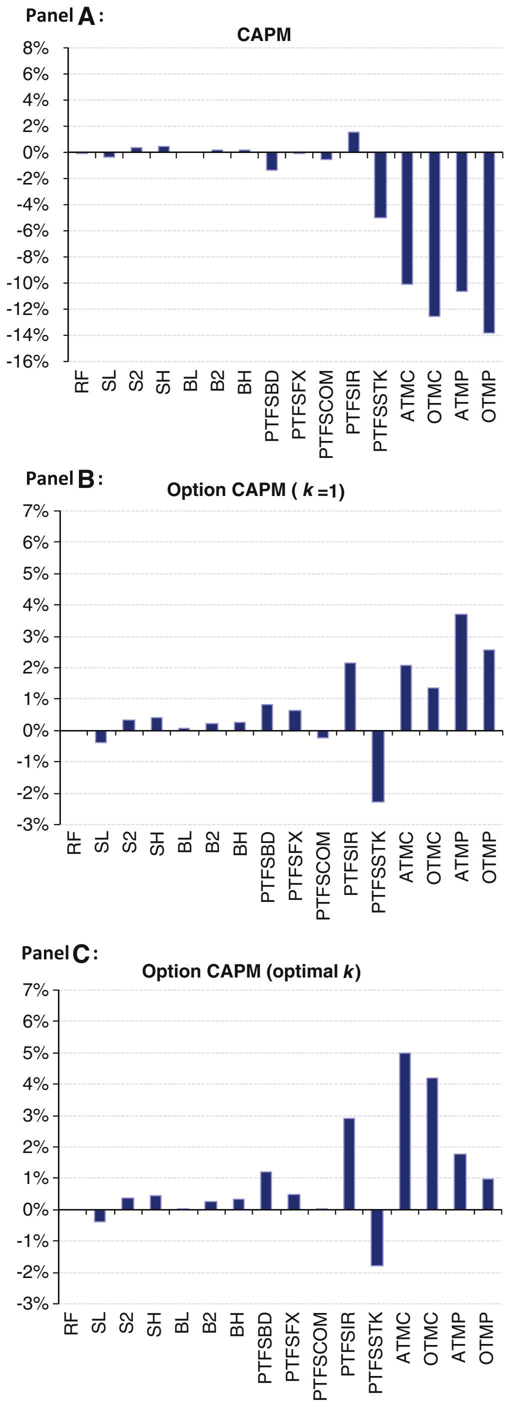


Fig. 1 Pricing errors for unconditional models. *Note* Data are monthly gross returns of the T-Bill, the six Fama–French portfolios, the five Fung and Hsieh Primitive Trend-Following Strategy (PTFS) portfolios, and the four Agarwal and Naik option portfolios. Data start in January 1994 and ends in June 2008 for a total of 174 monthly observations. RF is the risk-free rate proxied by the return on the T-Bill. The six Fama–French are denoted by SL, S2, SH, BL, B2 and BH where the S and B stand for "Small" and "Big" sized firms and L,2,H stand for "Low", "Medium" and "High" book-to-market ratio. The five Fung and Hsieh PTFS portfolios are denoted by PTFSBD, PTFSFX, PTFSCOM, PTFSIR, and PTFSSTK where PTFSBD is the return of PTFS bond lookback straddle, PTFSFX is the return on the currency lookback straddle, PTFSCOM is the return on the commodity lookback straddle, PTFSIR is the return on the short-term interest rate lookback straddle, and PTFSSTK is the return on the stock index lookback straddle. Finally, the four Agarwal and Naik option portfolios are denoted by ATMC, OTMC, ATMP, and OTMP where ATM stands for "at-the-money", OTM stands for "out-of-the-money", C stands for "Call" option, and P stands for "Put" option

lookback straddle strategy on the short-term rate, which increases to 2.16% per month from a level of 1.56%.

As noted in Table 2, optimally estimating the moneyness of the option term of the SDF in Eq. 20 helps to reduce the HJ-distance. Yet the results in Panel C of Fig. 1 suggest that such a reduction is not uniform across all pricing errors. For example, the model errors for the Agarwal–Naik put portfolios are smaller (by around a half of the errors of the option CAPM with fixed strike), while the errors on the call portfolios are substantially higher. Similarly, the error of the Fung–Hsieh PTFS portfolio on the stock index decreases (again if compared to the case of the option CAPM with fixed strike), at the same time that error on the lookback straddle strategy on the short-term rate increases to 2.92% per month. Again, the errors for the Fama–French portfolios remain at levels similar to those previously estimated.

Notice also that, in spite of reducing the pricing errors of non-linear assets, the unconditional option CAPM, with either fixed and estimated strike, does not help to solve the book-to-market (B/M) effect as the two higher B/M portfolios (SH and BH) have larger pricing errors than the other portfolios.

5.3 Pricing errors for conditional models

Figures 2 and 3 report the errors for the conditional models where the factors are scaled by the short-term interest rate and the NBER recession dummy respectively. The results emerging from these figures are similar to those for the unconditional models. First, all models do well when pricing the return of the Treasury-Bill. Second, for a given choice of the instrument, all three models provide a similar fit of the Fama–French portfolios. Third, the CAPM have difficulties in pricing the Agarwal–Naik portfolios and the Fung–Hsieh PTFS on the stock index and the short-term interest rate. Four, including an option term in the SDF proxy delivers an overall reduction of the pricing errors. Fifth, optimally estimating the moneyness of the option in the non-linear SDF reduces the HJ-distance, but such a reduction does not come from a uniform reduction across all pricing errors. In this way, the Agarwal–Naik call portfolios, and the Fung–Hsieh straddle portfolio on the short-term interest rate get their pricing errors increased.

6 Estimating hedge fund performance

The ultimate test for investors remains a positive value, that is, whether the value of the cash flows generated by a fund, net of all fees, is greater than the amount entrusted to the fund manager. In this section, we use the estimated SDF proxies to find which funds display a positive performance. As noted in Sect. 2 , a fund displaying a positive alpha indicates the existence of a manager who possesses investment information or skills such that he can generate superior portfolio returns. Table 3 presents estimated alphas according to Eq. 17 for each one of the Hedge Fund categories as well as p-values for the null hypothesis that these are zero against the alternative hypothesis

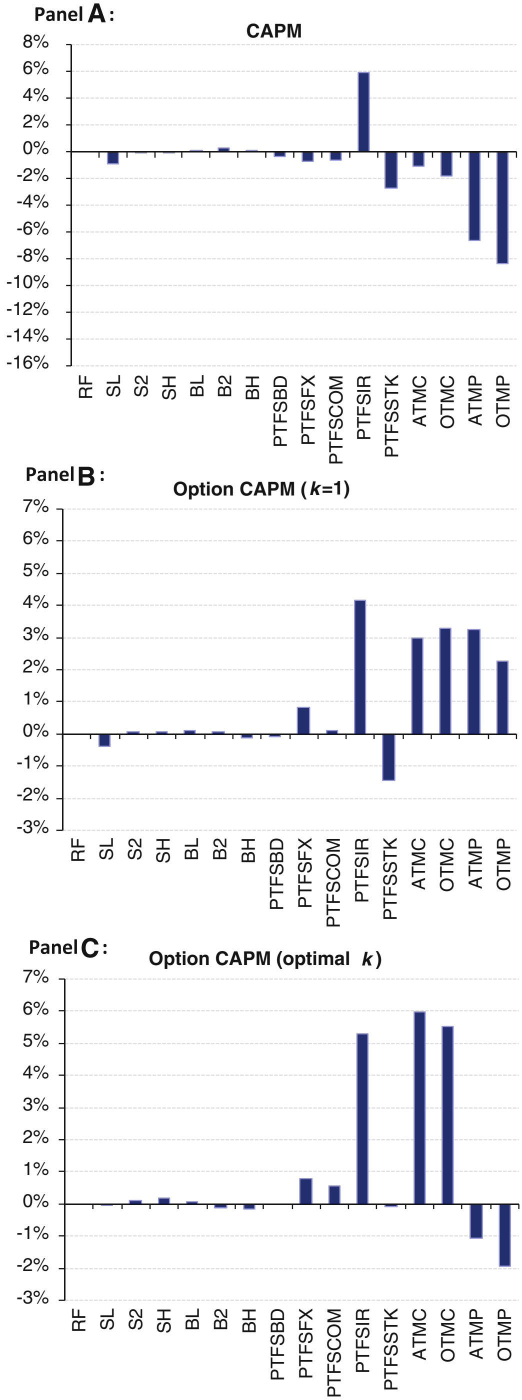
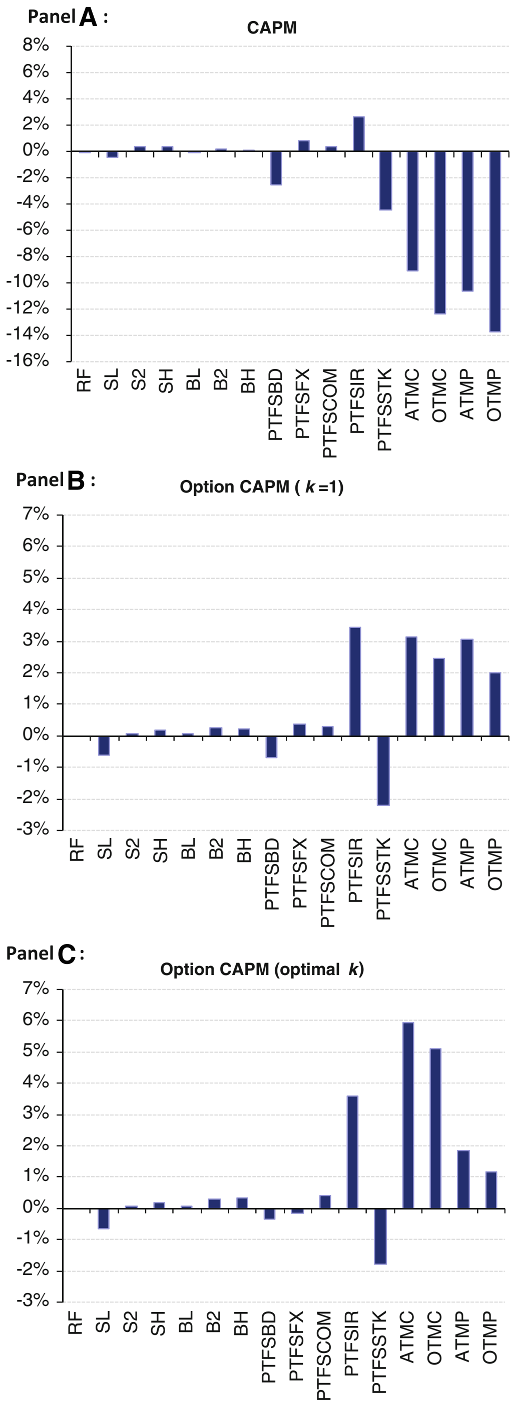


Fig. 2 Pricing errors for models using the short-term interest rate as the instrument panel A. *Note* Data are monthly gross returns of the T-Bill, the six Fama–French portfolios, the five Fung and Hsieh Primitive Trend-Following Strategy (PTFS) portfolios, and the four Agarwal and Naik option portfolios. Data start in January 1994 and ends in June 2008 for a total of 174 monthly observations. RF is the risk-free rate proxied by the return on the T-Bill. The six Fama–French are denoted by SL, S2, SH, BL, B2 and BH where the S and B stand for "Small" and "Big" sized firms and L,2,H stand for "Low", "Medium" and "High" book-to-market ratio. The five Fung and Hsieh PTFS portfolios are denoted by PTFSBD, PTFSFX, PTFSCOM, PTFSIR, and PTFSSTK where PTFSBD is the return of PTFS bond lookback straddle, PTFSFX is the return on the currency lookback straddle, PTFSCOM is the return on the commodity lookback straddle, PTFSIR is the return on the short-term interest rate lookback straddle, and PTFSSTK is the return on the stock index lookback straddle. Finally, the four Agarwal and Naik option portfolios are denoted by ATMC, OTMC, ATMP, and OTMP where ATM stands for "at-the-money", OTM stands for "out-of-the-money", C stands for "Call" option, and P stands for "Put" option

Fig. 3 Pricing errors for models using the NBER dummy as the instrument. *Note* Data are monthly gross returns of the T-Bill, the six Fama–French portfolios, the five Fung and Hsieh Primitive Trend-Following Strategy (PTFS) portfolios, and the four Agarwal and Naik option portfolios. Data start in January 1994 and ends in June 2008 for a total of 174 monthly observations. RF is the risk-free rate proxied by the return on the T-Bill. The six Fama–French are denoted by SL, S2, SH, BL, B2 and BH where the S and B stand for "Small" and "Big" sized firms and L,2,H stand for "Low", "Medium" and "High" book-to-market ratio. The five Fung and Hsieh PTFS portfolios are denoted by PTFSBD, PTFSFX, PTFSCOM, PTFSIR, and PTFSSTK where PTFSBD is the return of PTFS bond lookback straddle, PTFSFX is the return on the currency lookback straddle, PTFSCOM is the return on the commodity lookback straddle, PTFSIR is the return on the short-term interest rate lookback straddle, and PTFSSTK is the return on the stock index lookback straddle. Finally, the four Agarwal and Naik option portfolios are denoted by ATMC, OTMC, ATMP, and OTMP where ATM stands for "at-the-money", OTM stands for "out-of-the-money", C stands for "Call" option, and P stands for "Put" option



that they are positive. Such a one-sided hypothesis is tested using the (sup) t-statistic as explained in Sect. 3.[19]

6.1 Overall hedge fund industry index

As noted in the first line of Table 3, the overall hedge fund industry index presents a positive and significant performance regardless of the model used to estimate the portfolio performance. The industry alpha is close to 0.40% per month, and, while significant, such a misspricing is small if compared to the pricing errors of the Agarwal–Naik portfolios (see Figs. 1, 2, 3).

6.1.1 C1-Convertible arbitrage

When looking to unconditional models, the convertible arbitrage strategy presents a positive and significant performance, and the estimated alpha is close to 0.30% per month. While similar numbers are found when looking at the estimates obtained using the conditional model that uses the NBER dummy as the instrument, when we scale factors by the short-term rate and include a non-linear term in the SDF, we find that the performance of the convertible arbitrage strategy decreases and becomes only marginally significant (i.e. at the 10% level).

6.1.2 C2-Dedicated short-Bias

This category provides positive albeit statistically insignificant alpha. When looking to conditional models that scale factors by the short-term rate we find that this strategy provides positive value. In the case of the conditional CAPM we find a positive and significant alpha of 0.45%. However, performance decreases to 0.20% (significant at the 10% level) and 0.09% (significant at the 5% level) as soon as we allow for non-linearities in the SDF, and optimally estimate the strike of the option.

6.1.3 C3-Emerging markets

Non-linearities are especially important when computing the performance of the Emerging Markets strategy. In this case, ignoring the option term in the SDF would result in a positive performance estimate when, in fact, non-linear models suggest that this performance is negative. Moreover, when we use the short-term rate as a measure of public information we find that, even without accounting for these non-linearities, the estimated alpha is negative.

[19] As in Farnsworth et al. (2002), we estimated the joint system in Eq. 7 separately for each fund. As shown by these authors, such an approach is not restrictive and, in fact, it is asymptotically equivalent to joint estimation of all funds at the same time.

Table 3 Performance evaluation of hedge fund strategies

	Unconditional model			Short-term rate model			NBER dummy model		
		Option CAPM			Option CAPM			Option CAPM	
	CAPM	$k = 1$	Optimal	CAPM	$k = 1$	Optimal	CAPM	$k = 1$	Optimal
Hedge fund industry	0.391	0.353	0.377	0.413	0.424	0.417	0.427	0.312	0.307
	[0.003]	[0.035]	[0.049]	[0.034]	[0.006]	[0.017]	[0.001]	[0.046]	[0.068]
C1– Convertible arbitrage	0.298	0.320	0.282	0.358	0.126	0.172	0.293	0.306	0.279
	[0.011]	[0.033]	[0.040]	[0.002]	[0.077]	[0.102]	[0.012]	[0.015]	[0.030]
C2– Dedicated short bias	0.190	0.195	0.210	0.452	0.202	0.091	0.103	0.177	0.224
	[0.189]	[0.245]	[0.163]	[0.027]	[0.097]	[0.046]	[0.328]	[0.223]	[0.183]
C3– Emerging markets	0.232	−0.027	−0.095	−0.369	−0.101	−0.201	0.301	−0.057	−0.145
	[0.164]	[0.458]	[0.470]	[0.956]	[0.838]	[0.889]	[0.150]	[0.549]	[0.557]
C4– Equity market neutral	0.435	0.487	0.490	0.411	0.293	0.309	0.456	0.501	0.482
	[0.000]	[0.000]	[0.000]	[0.000]	[0.002]	[0.001]	[0.000]	[0.000]	[0.000]
C5– Event driven	0.462	0.278	0.198	0.394	0.295	0.154	0.468	0.272	0.161
	[0.000]	[0.094]	[0.070]	[0.031]	[0.075]	[0.101]	[0.001]	[0.088]	[0.076]
C6– Fixed income arbitrage	0.128	0.088	0.099	0.083	0.107	0.088	0.090	0.083	0.130
	[0.102]	[0.259]	[0.269]	[0.480]	[0.300]	[0.345]	[0.287]	[0.303]	[0.237]
C7– Global macro	0.729	0.742	0.799	0.846	0.867	0.774	0.710	0.540	0.529
	[0.000]	[0.007]	[0.004]	[0.025]	[0.000]	[0.001]	[0.004]	[0.037]	[0.057]
C8– Long/short equity hedge	0.363	0.412	0.418	0.304	0.285	0.333	0.324	0.211	0.172
	[0.008]	[0.041]	[0.056]	[0.096]	[0.116]	[0.125]	[0.012]	[0.126]	[0.144]
9– Managed futures	0.361	0.758	0.891	−0.053	0.328	0.425	0.376	0.094	0.159
	[0.084]	[0.010]	[0.000]	[0.764]	[0.300]	[0.287]	[0.089]	[0.356]	[0.349]

This table presents estimated alphas for each one of the Hedge Fund categories. Similarly, p-values for the null hypothesis that these are zero, $H_0 : \alpha = 0$, against the alternative hypothesis that they are positive, $H_1 : \alpha > 0$, are presented in squared brackets. The sample period is January 1994 to June 2008 (174 observations).

6.1.4 C4-Equity-market neutral

As in the case of the overall hedge fund industry, the Equity Market Neutral strategy presents a positive and significant performance regardless the model used to compute the alpha. Only when accounting for the non-linearities in the short-term rate conditional model, we observe a fall in the performance from around 0.40% per month to around 0.30%. Yet, as mentioned before, these numbers are still statistically different from zero at the conventional significance levels.

6.1.5 C5-Event driven

This strategy is another case where non-linearities have to be taken into account in order to arrive to the true picture of the industry. Alphas computed according from the (either unconditional or conditional) CAPM are all positive, significant, and found to be between 0.40 and 0.50% per month. Yet when we include an option term in the SDF, such a performance is reduced to around 0.28% and only significant at the 10% level. Allowing for the strike of the option to be optimally estimated further decreases the estimated alpha to a level of 0.16%, a number that, again, is only significant at the 10% level.

6.1.6 C6-Fixed income arbitrage

This category tends to present the lowest performance across all categories and, while the estimated alpha is positive for all possible models, it cannot be rejected that the estimated performance is equal to zero at the conventional significance levels.

6.1.7 C7-Global macro

As in the case of the Overall Hedge Fund Industry and the Equity Market Neutral strategy, this strategy presents a positive and significant alpha. This result is robust to the choice of model used to compute the performance. Estimated alphas range from 0.53% per month (NBER recession dummy model with optimally estimated strike) to 0.87% (short-term rate model with fixed strike).

6.1.8 C8-Long-short equity hedge

Accounting for conditional information seems to be important when estimating the performance of this strategy. Alphas computed from unconditional models are all positive and significant at the conventional levels. However, when we allow factors to vary according to our two instruments, either the short-term rate or the NBER recession dummy, performance drops at the time that the p-value for the null hypothesis that the true alpha is zero increases over the 10% level.

6.1.9 C9-Managed futures

Again, non-linearities and public information are key elements that one has to consider to obtain a true picture of the managed future funds. The alphas computed from unconditional models are all positive and significantly different from zero. It is also worth mentioning that, in contrast to the results for other strategies, ignoring the option term in the SDF proxy leads to a large underestimation of the performance of the strategy. More important is that when including conditional information into the analysis, the estimated alpha sharply falls becoming even negative when using the short-rate CAPM to compute the performance of the fund.

In summary, if we use as a benchmark the option CAPM where factors are scaled by the short-term rate (either with fixed or estimated strike), the only models for which we cannot reject that the HJ-distance is equal to zero, we only find evidence of positive performance for the overall hedge fund index, the equity-market neutral as well as the global macro strategy.

7 Final remarks

The objective of this paper is to find whether hedge funds provide a positive performance to investors, that is, whether the value of the cash flows generated by a fund, net of all fees, is greater than the amount entrusted to the fund manager. To do so, we focus on an asset pricing model that is characterized by a piecewise-linear SDF in order to better able to capture the non-linear structures and option-like features exhibited by hedge fund returns (see e.g. Agarwal and Naik 2004; Diez de los Rios and Garcia 2010). We estimate this SDF using the generalized method of moments by minimizing the so-called Hansen–Jagannathan distance to then test whether our proposed non-linear SDF also prices the funds. Our results show that, once non-linearities and public information are taken into account, there is only evidence of positive performance for the overall hedge fund index, the equity-market neutral and the global macro strategy. We also find strong evidence in favour of the existence of non-linearities in the stochastic discount factor.

Several interesting issues are left for further research. In this paper we have only focused on an asset pricing model that is characterized by a piecewise-linear SDF. Yet there are several other asset pricing models such as those in Harvey and Siddique (2000), Dittmar (2002), or Vanden 2006 that generate non-linear pricing kernels. These models are natural alternatives to the option CAPM, and further research on which models provides the best fit for the set of benchmark assets is still needed.

Since the technical conditions needed for the bootstrap distribution to achieve a higher rate of convergence to the sampling distribution than the first-order asymptotic approximation have not been verified in our context, it would be interesting to run a Monte Carlo experiment to assess the finite-sample properties of the estimators proposed in this paper.

Another open question is whether the model that uses the short-term interest rate as the instrument is still able to price all the benchmark assets when one imposes a non-negativity constraint when estimating the SDF model.

Finally, an area that deserves further investigation is the use of individual funds data to obtain hedge fund indexes that correct for the back-filling and liquidation biases mentioned in the main text of the paper, and to obtain a more comprehensive picture of the hedge fund industry.

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Appendices

A. Bootstrap distributions

To obtain the bootstrap distribution of the test statistics of interest, we will first assume that the true SDF is linear in the set of factors $\mathbf{f}_t^0$:

$$y_{t+1} = \lambda_0^0 + \mathbf{f}_{t+1}^{0\prime}\boldsymbol{\lambda}_1^0 = \mathbf{x}_{t+1}^{0\prime}\boldsymbol{\lambda}^0, \tag{24}$$

where $\boldsymbol{\lambda}^0 = \left(\lambda_0^0, \boldsymbol{\lambda}_1^{0\prime}\right)'$ and $\mathbf{x}_{t+1}^{0\prime} = \left(1, \mathbf{f}_{t+1}^{0\prime}\right)'$. Superscript 0 denotes the true pricing kernel. For example, we could assume that the true asset pricing model is the unconditional CAPM and, therefore, $\mathbf{f}_{t+1}^0 = R_{m,t}$. We could similarly assume a conditional version of the CAPM so $\mathbf{f}_{t+1}^0 = \left(z_t, R_{m,t}, z_t R_{m,t}\right)'$ where z_t is a scaling variable.

Before presenting our bootstrap procedures, we introduce some helpful notation. In particular, let $\mathbf{Y}_t^0 = \left[\mathbf{f}_t^{0\prime}, \mathbf{R}_{t+1}'\right]'$ and define its mean and variance as

$$\mu = E\left[\mathbf{Y}_t^0\right] \equiv \begin{bmatrix} \mu_1^0 \\ \mu_2 \end{bmatrix}, \tag{25}$$

$$\mathbf{V} = \operatorname{Var}\left[\mathbf{Y}_t^0\right] \equiv \begin{bmatrix} \mathbf{V}_{11}^0 & \mathbf{V}_{12}^0 \\ \mathbf{V}_{21}^0 & \mathbf{V}_{22} \end{bmatrix}. \tag{26}$$

To obtain the bootstrap distribution of the test statistics of interest, we proceed as follows:

1. We first obtain an estimate $\widehat{\boldsymbol{\lambda}}^0$ under the null model by minimizing the HJ-distance as shown in Sect. 2.
2. Second, we estimate a linear factor model on the returns

$$\mathbf{R}_t = \alpha + \beta \mathbf{f}_t^0 + \varepsilon_t \tag{27}$$

   where $\beta = \mathbf{V}_{21}^0 \left(\mathbf{V}_{11}^0\right)^{-1}, \alpha = \mu_2 - \beta\mu_1^0$, and ε_t is i.i.d. with $E\left(\varepsilon_t\right) = \mathbf{0}_N$ and $E\left(\varepsilon_t | \mathbf{f}_t\right) = \mathbf{0}_N$. This way, we obtain coefficient estimates $\widehat{\alpha}$ and $\widehat{\beta}$ as well as a time series of estimated residuals $\widehat{\varepsilon}_t = \mathbf{R}_{t+1} - \widehat{\alpha} - \widehat{\beta}\mathbf{f}_t^0$.
3. Using as a baseline an i.i.d bootstrap, we draw, as in Kosowski et al. (2006), a sample with replacement from the residuals that are saved in the second step above to

create a pseudo–time series of resampled residuals $\{\widehat{\varepsilon}_t^b\}$ where b is an index for the bootstrap number (so $b = 1$ for bootstrap resample number 1). The original chronological ordering of the factor returns $\mathbf{f}_t = R_{M,t}$ is left unaltered.

4. Next, we construct a time series of pseudo returns imposing the null hypothesis that the true SDF is linear (equation above). In particular, note that when the model is correctly specified we have,

$$E\left(\mathbf{R}_{t+1}\mathbf{x}_{t+1}^{0\prime}\boldsymbol{\lambda}^0\right) = \mathbf{1}_{n+1}$$
$$E(\mathbf{R}_{t+1})\boldsymbol{\lambda}_0^0 + E\left(\mathbf{R}_{t+1}\mathbf{f}_{t+1}^{0\prime}\right)\boldsymbol{\lambda}_1^0 = \mathbf{1}_{n+1}$$
$$\mu_2\boldsymbol{\lambda}_0^0 + \left(\mathbf{V}_{21}^0 + \mu_2\mu_1^{0\prime}\right)\boldsymbol{\lambda}_1^0 = \mathbf{1}_{n+1}$$
$$\mu_2 = \frac{1}{\left(\boldsymbol{\lambda}_0 + \mu_1^{0\prime}\boldsymbol{\lambda}_1^0\right)}\left(\mathbf{1}_{n+1} - \mathbf{V}_{21}^0\boldsymbol{\lambda}_1^0\right)$$

and substituting this expression into $\alpha = \mu_2 - \beta\mu_1^0$ gives:

$$\alpha = \frac{1}{\left(\boldsymbol{\lambda}_0 + \mu_1^{0\prime}\boldsymbol{\lambda}_1^0\right)}\left(\mathbf{1}_n - \mathbf{V}_{21}^0\boldsymbol{\lambda}_1\right) - \mathbf{V}_{21}^0\left(\mathbf{V}_{11}^0\right)^{-1}\mu_1^0$$

Therefore, we can construct a time series of pseudo returns under the null as

$$\mathbf{R}_t^b = \frac{1}{\left(\widehat{\boldsymbol{\lambda}}_0 + \widehat{\mu}_1^{0\prime}\widehat{\boldsymbol{\lambda}}_1^0\right)}\left(\mathbf{1}_n - \widehat{\mathbf{V}}_{21}^0\widehat{\boldsymbol{\lambda}}_1^0\right) + \widehat{\mathbf{V}}_{21}^0\left(\widehat{\mathbf{V}}_{11}^0\right)^{-1}(\mathbf{f}_t^0 - \widehat{\mu}_1^0) + \widehat{\varepsilon}_t^b \quad (28)$$

where, $\widehat{\boldsymbol{\lambda}}^0$ is the estimate of $\boldsymbol{\lambda}^0$ obtained in the first step, and $\widehat{\mu}$ and $\widehat{\mathbf{V}}$ are the sample versions of μ and $\mathbf{V}$ in (25) and (26). Note that this sequence of artificial returns has pricing errors that are zero by construction, and therefore, the corresponding HJ-distance also is equal to zero.

5. Finally, we use this time-series of pseudo returns $\{\mathbf{R}_t^b\}$ to estimate the option CAPM model and save the corresponding supWald test for the null-hypothesis that the model is linear, $W_T^{\lambda_2}$; the supWald test for the null-hypothesis that all pricing errors are equal to zero, $W_T^{\mathbf{g}}$; and the sup$t$ -statistic for the the null-hypothesis that the fund provides a null performance t_T^{α}. We also save the corresponding squared HJ-distance evaluated at the optimal value of k.

Repeating the above steps 3–5 for all bootstrap iterations, $b = 1, \ldots, 2000$, we then build the distribution of our test statistics that result purely from sampling variation while imposing the null that the true model is linear.

B. Brief definitions of hedge fund categories

The following definitions are based on the book by Lhabitant (2004).

C1 Convertible arbitrage

A typical strategy in this category is to be long in the convertible bond and short in the common stock of the same company. Profits are generated from both positions. The principal is usually protected from market fluctuations.

C2 Dedicated short bias

Dedicated short hedge funds seek to profit from a decline in the value of stocks by taking short positions. These funds are rare nowadays, since they migrated to the long/short category, where they still have a systematic short bias.

C3 Emerging markets

These funds take positions in all types of securities in emerging markets around the world. Investments in emerging market equities are primarily long, since many emerging markets do not allow short selling and no viable futures markets exist to hedge market risk.

C4 Equity market neutral

This investment strategy aims at balancing long and short positions to ensure a negligible market exposure in a broad sense. A fund may be neutral to a specific exchange rate, a stock index, a series of interest rates, or other factors.

C5 Event driven

This strategy aims at making profits by using price movements related to special pending events such as mergers, liquidations, bankruptcies, or reorganizations. In risk arbitrage, the hedge fund manager usually invests long in the stock of the company being acquired and short in the stock of the acquiring company.

C6 Fixed-income arbitrage

The goal is to exploit price anomalies between related interest rate securities, such as interest rate swaps, US and non-US government bonds, and mortgage-backed securities.

C7 Global macro

Global macro funds do not hedge anything. They make directional bets based on their forecasts of market directions according to economic trends or particular events. They are not specialized, and carry long and short positions in any of the major world capital or derivative markets. The portfolios include stocks, bonds, currencies, and commodities. Most funds invest globally in both developed and emerging markets.

C8 Long-short equity

Long/short strategies involve the combined purchase and sale of two securities. The main source of return comes from the spread in performance between the stocks on the long side (which should appreciate in value) and the shorted stocks (which should decrease in value). The strategies can be based on value, growth, or size.

C9 Managed futures

These funds, often referred to as commodity trading advisers (CTAs), invest in financial and commodity futures markets and currency markets around the world. A large proportion are trend followers (buy in an up market and sell in a down market). Others use discretionary (judgmental) or systematic (based on technical information) strategies.

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