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Dependence structure and extreme comovements in international equity and bond markets

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ABSTRACT

Common negative extreme variations in returns are prevalent in international equity markets. This has been widely documented with statistical tools such as exceedance correlation, extreme value theory, and Gaussian bivariate GARCH or regime-switching models. We point to limits of these tools to characterize extreme dependence and propose an alternative regime-switching copula model that includes one normal regime in which dependence is symmetric and a second regime characterized by asymmetric dependence. We apply this model to international equity and bond markets, to allow for inter-market movements. Empirically, we find that dependence between international assets of the same type is strong in both regimes, especially in the asymmetric one, but weak between equities and bonds, even in the same country.

1. Introduction

There is ample evidence that negative returns are more dependent than positive returns in international equity markets. This phenomenon known as asymmetric dependence has been reported by many previous studies including Erb et al. (1994), Longin and Solnik (2001), Ang and Bekaert (2002), Ang and Chen (2002), Das and Uppal (2004), Patton (2004), and references therein. This asymmetric dependence has important implications for portfolio allocation, but to appreciate its full actual effects on portfolio diversification, stocks and bonds have to be considered together, both at the domestic and international levels to allow for inter-market movements.[1] Models of extreme dependence in international stock and bond markets are mainly missing in the literature. This is due mainly to the fact that measuring and modeling asymmetric dependence remains a challenge.

Previous studies relied on the concept of exceedance correlation, correlation computed for returns above or below a certain threshold, to investigate the dependence structure between financial returns.[2] Boyer et al. (1999) and Forbes and Rigobon (2002) remark that correlations estimated conditionally on high or low returns or volatility suffer from some conditioning bias. Correlation asymmetry may therefore appear spuriously if these biases are not accounted for. To avoid these problems, Longin and Solnik (2001) use extreme value theory (EVT) by focusing on the asymptotic value of exceedance correlation.[3] The benefit of EVT resides in the fact that the asymptotic result holds regardless of the distribution of returns. By the same token, as emphasized by Longin and Solnik (2001), EVT

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[1] Patton (2004) finds that the knowledge of asymmetric dependence leads to gains that are economically significant, while Ang and Bekaert (2002), in a regime switching setup, argue that the costs of ignoring the difference between regimes of high and low dependence are small, but increase with the possibility to invest in a risk-free asset.

[2] The exceedance correlation between two series of returns is defined as the correlation for a subsample in which the returns of both series are simultaneously lower (or greater) than the corresponding thresholds θ_1 and θ_2. Formally, exceedance correlation of variables X and Y at thresholds θ_1 and θ_2 is expressed by $Ex\_corr(Y,X;\ \theta_1,\theta_2) = \begin{cases} corr(X,Y|X \leqslant \theta_1,\ Y \leqslant \theta_2), \text{ for } \theta_1 \leqslant 0 \text{ and } \theta_2 \leqslant 0 \\ corr(X,Y|X \geqslant \theta_1,\ Y \geqslant \theta_2), \text{ for } \theta_1 \geqslant 0 \text{ and } \theta_2 \geqslant 0 \end{cases}$. Longin and Solnik (2001) use $\theta_1 = \theta_2 = \theta$, while Ang and Chen (2002) use $\theta_1 = (1+\theta)\overline{X}$ and $\theta_2 = (1+\theta)\overline{Y}$, where $\overline{X}$ and $\overline{Y}$ are the means of Y and X respectively.

[3] Extreme value theory (EVT) is used to characterize the distribution of a variable conditionally to the fact that its values are beyond a certain threshold, and the asymptotic distribution is obtained when this threshold tends to infinity. Hartmann et al. (2004) also use extreme-value analysis to capture the dependence structure between stock and bond returns for pairs of the G5 countries.

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cannot help to determine if a given return-generating process is able to reproduce the extreme asymmetric exceedance correlation observed in the data.

To overcome this shortcoming, we propose a model based on copulas that allows for tail dependence in lower returns and keeps tail independence for upper returns as suggested by the findings of Longin and Solnik (2001). Copulas are functions that build multivariate distribution functions from their unidimensional marginal distributions.[4] The tail dependence coefficient can be seen as the probability of the worst event occurring in one market given that the worst event occurs in another market. Contrary to exceedance correlation, the estimation of the tail dependence coefficient is not subject to the problem of choosing an appropriate threshold and the use of extreme value distributions such as the Pareto distribution. Another difference is that tail dependence is completely defined by the dependence structure and is not affected by variations in marginal distributions.

The disentangling between marginal distributions and dependence helps overcoming the curse of dimensionality associated with the estimation of models with several variables. For example, in multivariate GARCH models, the estimation becomes intractable when the number of series being modeled is high. The CCC of Bollerslev (1990), the DCC of Engle (2002), and the RSDC of Pelletier (2006) deal with this problem by separating the variance–covariance matrix in two parts, one part for the univariate variances of the different marginal distributions, another part for the correlation coefficients. This separation allows them to estimate the model in two steps, first the marginal parameters on each individual series then the correlation parameters. Copulas offer a tool to generalize this separation while extending the linear concept of correlation to nonlinear dependence.

Thanks to the tail dependence formulation of asymptotic dependence, we show analytically that the multivariate GARCH or regime switching (RS) models with Gaussian innovations that have been used to address asymmetric dependence issues (see Ang and Bekaert, 2002; Ang and Chen, 2002) cannot in fact reproduce extreme asymmetric dependence. The key point is that these classes of models can be seen as mixtures of symmetric distributions and cannot produce asymptotically asymmetric dependence. The asymmetry produced by these models at finite distance disappears asymptotically. When we go far in the tails, we obtain a similar dependence for the upper and lower tails. Moreover, the asymmetry in RS models comes from the asymmetry created in the marginal distributions with regime switching in the mean. Hence it is not separable from the marginal asymmetry or skewness.[5] This is a fundamental issue that also affects the statistical extreme-value analysis that have been conducted to study extreme dependence.

We use our regime-switching copula model to investigate the dependence structure between international equity and bond markets. The model allows for a switching between a normal state where markets will be linearly and symmetrically correlated and an asymmetric dependence state to capture common crashes. In a normal regime it is difficult to make a difference between the level of dependence for joint positive moves and joint negative moves. When the economy is in the asymmetric regime, even with a stable correlation, a downside move in one market will increase the probability of a similar event in another market. The rise in the level of dependence during market downturns is characterized by asymmetry in the dependence structure. This regime can be interpreted as contagion since bad news spread quickly between markets. This crash dependence can coexist with low correlation and implies a reduction of an apparent diversification benefit.

We separately analyze dependence between the two leading markets in North-America (US and Canada) and two major markets of the Euro zone (France and Germany). Our empirical analysis shows that dependence between international assets of the same type is strong in both the symmetric and the asymmetric regimes, while dependence between equities and bonds is low even in the same country. Another finding is that the presence of a regime with extreme asymmetric dependence makes the correlation in the normal regime differ from the unconditional correlation. We also provide some evidence that exchange rate volatility seems to contribute to asymmetric dependence. With the introduction of a fixed exchange rate the dependence between France and Germany becomes less asymmetric and more normal than before. High exchange rate volatility is associated with a high level of asymmetry. These results are consistent with those of Cappiello et al. (2006) who find an increase in correlation after the introduction of the Euro currency.[6]

The rest of this paper is organized as follows. Section 2 reformulates the empirical facts about exceedance correlation in terms of tail dependence and shows how classical GARCH or regime-switching models fail to capture these facts. In Section 3 we develop a model with two regimes that clearly disentangles dependence from marginal distributional features and allows asymmetry in extreme dependence. As a result, we obtain a model with four variables that features asymmetry and a flexible dependence structure. Empirical evidence on the dependence structure is examined in Section 4, while conclusions are drawn in Section 5.

2. Extreme asymmetric dependence and modeling issues

In this section we present empirical facts about exceedance correlation in international equity market returns put forward by Longin and Solnik (2001) and the related literature. We next argue that these facts can be equivalently reformulated in terms of tail dependence. The latter formulation will allow us to explain why classical return-generating processes such as GARCH and regime-switching models based on a multivariate normal distribution fail to reproduce these empirical facts.

2.1. Empirical facts

Longin and Solnik (2001) investigate the structure of correlation between various equity markets in extreme situations by testing the equality of exceedance correlations, one obtained under a joint normality assumption and the other one computed using EVT. For the latter distribution, they model the marginal distributions of equity index returns with a generalized Pareto distribution (GPD) and capture dependence through a logistic function. Ang and Chen (2002) develop a test statistic based on the difference between exceedance correlations computed from the data and those obtained from GARCH or RS models.[7]

[4] The theory of this useful tool dates back to Sklar (1959) and a clear presentation can be found in Nelsen (1999). Well designed to analyze nonlinear dependence, copulas were initially used by statisticians for nonparametric estimation and measure of dependence of random variables (see Genest and Rivest, 1993 and references therein).

[5] Ang and Chen (2002) conclude that even if regime-switching models perform best in explaining the amount of correlation asymmetry reflected in the data, these models still leave a significant amount of correlation asymmetry in the data unexplained.

[6] Some related research focus on the dependence structure of bond and equity in European markets (see Kim et al., 2006; Cappiello et al., 2006; Abad et al., 2010). Panchenko and Wu (2009) investigate stock and bond return time-varying comovement in emerging markets, while Markwat et al. (2009) and Kumar and Okimoto (2011) analyze international integration in a global perspective.

[7] They define a test statistic $H = \left[\sum_{i=1}^{N} \frac{1}{N}(\rho(\vartheta_i) - \hat{\rho}(\vartheta_i))^2\right]^{1/2}$ which is the distance between exceedance correlations obtained from the normal distribution $(\rho(\vartheta_1), \ldots, \rho(\vartheta_N))$ and exceedance correlations estimated from the data $(\hat{\rho}(\vartheta_1), \ldots, \hat{\rho}(\vartheta_N))$ for a set of N selected thresholds $\{\vartheta_1, \ldots, \vartheta_N\}$. In the same way they define H^- and H^+ by considering negative points for H^- and nonnegative points for H^+ such that $H^2 = (H^-)^2 + (H^+)^2$. They can therefore conclude to asymmetry if H^- differs from H^+.

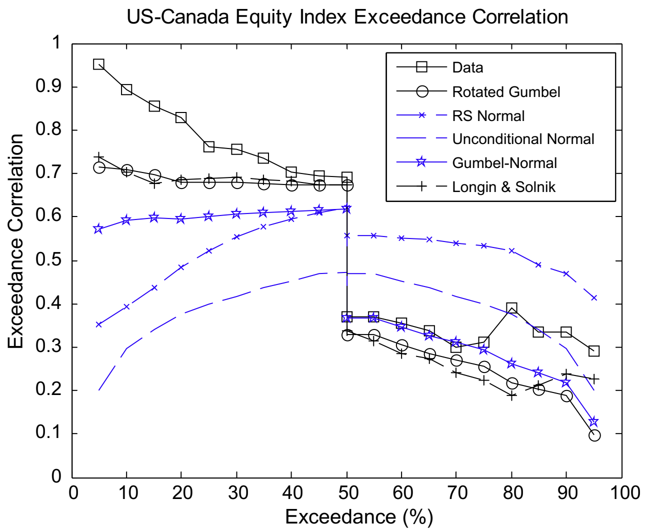

Fig. 1. Calculates correlations from US–Canada equity returns data for different values of the threshold θ, which is normalized. For θ less than 50% the correlation is calculated for the left tail and for θ greater than 50%, the correlation is calculated for the right tail. θ = 80% means that we calculate the correlation conditional on the 20% greatest observations for both US and Canadian equity returns, and θ = 10% means that we calculate the correlation conditional on the 10% lowest observations for both US and Canadian equity returns. The solid line with squares represents the exceedance correlations calculated directly from the data. For the Rotated Gumbel Copula with Gaussian Margins (Rotated Gumbel), the Normal Regime Switching Distribution (RS Normal), the Normal Distribution (Unconditional Normal), and the mixture of rotated Gumbel and Normal Copula (Gumbel–Normal) used in this paper, we first estimate the model and use the estimates to generate 50,000 Monte Carlo simulations to compute correlations. Longin and Solnik exceedance correlations are obtained by Longin and Solnik (2001) estimation method.

These two studies conclude that there exists asymmetry in exceedance correlation, that is large negative returns are more correlated than large positive returns. However, their results rely on choosing a set of thresholds for computing exceedance correlation and can only account for asymmetry at finite distance. Crashes are more in the nature of extreme events and involve measuring dependence for thresholds very far in the tail. Longin and Solnik (2001) confirm with an asymptotic test that exceedance correlation is positive and statistically different from zero for very large negative returns and not different from zero for very large positive returns. However they do not provide a model that is able to reproduce this fact. Ang and Chen (2002) as well as Ang and Bekaert (2002) find that regime-switching models can reproduce the asymmetry in exceedance correlation, but this result does not hold for extreme events as we will show later and the measured asymmetry amalgamates skewness in the marginal distributions and asymmetric dependence.

We illustrate these facts and the capacity of models to reproduce them in Fig. 1 with US and Canadian returns. We specify thresholds in term of quantiles: $\theta_1 = F_X^{-1}(\alpha)$ and $\theta_2 = F_Y^{-1}(\alpha)$ where F_X and F_Y are the cumulative distribution functions of Y and X respectively. Following Longin and Solnik (2001) and Ang and Chen (2002) exceedance correlations are symmetric if $Ex\_corr(Y,X;\theta_1,\theta_2) = Ex\_corr(Y,X;1-\theta_1,1-\theta_2);\alpha \in (0,1)$. Correlations of return exceedances exhibit the typical shape put forward in Longin and Solnik (2001) for the US equity market with various European equity markets. For the models, we chose to retain the multivariate normal, as a benchmark case to show that correlations go to zero as we move further in the tails, as well as a normal regime-switching model, as in Ang and Chen (2002). The latter model produces some asymmetry in correlations for positive and negative returns but not nearly as much as in the data. We also exhibit the exceedance correlations estimated with the procedure used by Longin and Solnik (2001). It is evidently much closer to the data. Finally, we also report the correlations obtained from a rotated Gumbel copula for the dependence function (see Appendix A for a definition), with Gaussian marginal distributions. The graph is very close to the Longin and Solnik (2001) one.

Since asymptotic exceedance correlation is zero for both sides of a bivariate normal distribution, Longin and Solnik (2001) interpreted these findings as rejection of normality for large negative returns and non-rejection for large positive returns. In the conclusion of their article, Longin and Solnik stress that their approach has the disadvantage of not explicitly specifying the class of return-generating processes that fail to reproduce these two facts. The difficulty in telling which model can reproduce these facts is the lack of analytical expressions for the asymptotic exceedance correlation and its intractability even for classical models such as Gaussian GARCH or regime-switching models. In order to investigate this issue, we introduce the concept of tail dependence. This will help us show analytically that some classes of models previously used in the literature cannot reproduce these asymmetries in extreme dependence and then propose a model that succeeds in doing so.

2.2. Tail dependence

To measure the dependence between an extreme event on one market and a similar event on another market, we define two dependence functions one for the lower tail and one for the upper tail, with their corresponding asymptotic tail dependence coefficients. For two random variables X and Y with cumulative distribution functions F_X and F_Y respectively, we call the lower tail dependence function (TDF) the conditional probability $\tau^L(\alpha) \equiv \Pr[X \leqslant F_X^{-1}(\alpha)|Y \leqslant F_Y^{-1}(\alpha)]$ for $\alpha \in (0,1/2]$ and similarly, the upper tail dependence function is $\tau^U(\alpha) \equiv \Pr[X \geqslant F_X^{-1}(1-\alpha)|Y \geqslant F_Y^{-1}(1-\alpha)]$.[8] The tail dependence coefficient (TDC) is simply the limit (when it exists) of this function when α tends to zero. More precisely *lower TDC* is $\tau^L = \lim_{\alpha\to 0}\tau^L(\alpha)$ and *upper TDC* is $\tau^U = \lim_{\alpha\to 0}\tau^U(\alpha)$. As in the case of joint normality, we have lower tail independence when $\tau^L = 0$ and upper tail independence for $\tau^U = 0$.

Compared to exceedance correlation used by Longin and Solnik (2001), Ang and Chen (2002), Ang and Bekaert (2002), and Patton (2004), a key advantage of TDF and corresponding TDCs is their invariance to modifications of marginal distributions that do not affect the dependence structure. Fig. 2 gives an illustration of this invariance. We simulate a bivariate Gaussian distribution $N(0,I_\rho)$, where I_ρ is the bi-dimensional matrix with standard deviations equal to one on the diagonal and a correlation coefficient ρ equal to 0.5. Both exceedance correlation and tail dependence measures show a symmetric behavior of dependence in extreme returns. However, when we replace one of the marginal distributions $N(0,1)$ by a mixture of normals, a $N(0,1)$ and a $N(4,4)$ with equal weights, and let the other marginal distribution and the dependence structure unchanged, the TDF remains the same while the exceedance correlation is affected. In fact, the correlation coefficient and the exceedance correlation are a function of the dependence structure and of the marginal distributions while the tail dependence is a sole function of the dependence structure, regardless of the marginal distributions. Another problem with asymptotic exceedance correlation estimation using extreme value is the sample bias since fewer data points are available when we

[8] In the literature (see Rodriguez, 2007 and references therein), only the limit of this function is considered. Here, we define the TDF for every $\alpha \in (0,1/2]$ to make a comparison with conditional correlation, which is also a function of a threshold. The tail dependence measure is also related to the concept of lower (upper) orthant dependence concept (see Denuit and Scaillet, 2004).

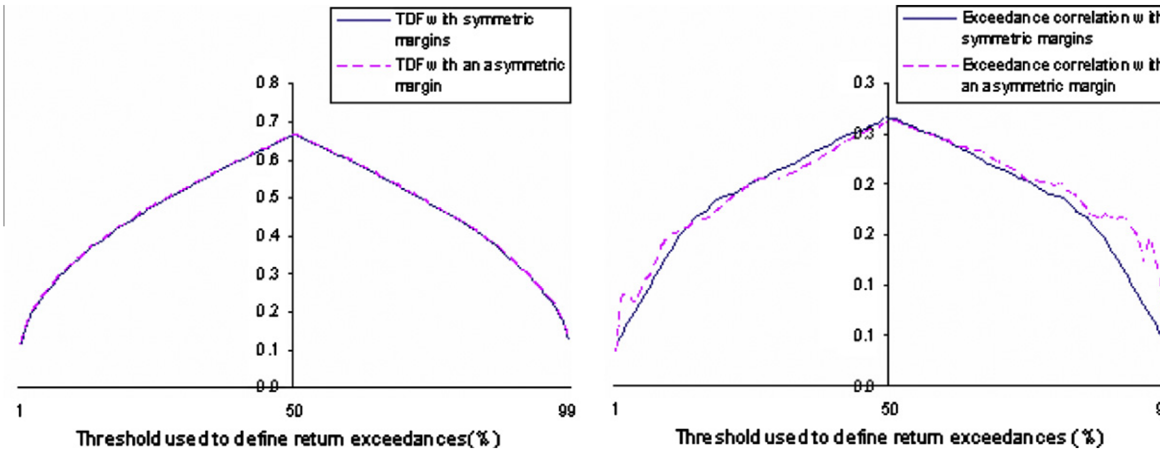

Fig. 2. Effect of marginal distribution asymmetry on the tail dependence function (TDF) and on exceedance correlation: first, we simulate a standard bivariate Gaussian distribution with correlation 0.5 and compute the TDF and exceedance correlation. Second, we create asymmetry in one marginal distribution by replacing the $N(0,1)$ by a mixture of $N(0,1)$ and $N(4,4)$ with equal weight.

move further into the tails of the distribution.[9] With tail dependence, the estimation can be done using all data points in the sample and the estimators of the tail coefficients are unbiased.[10]

By observing that for the logistic function used by Longin and Solnik (2001), the zero value for the asymptotic correlation coefficient is exactly equivalent to tail independence, we can reformulate their asymptotic result as follows: lower extreme returns are tail-dependent, while upper extreme returns are tail-independent.[11]

This reformulation presents at least two main advantages. Compared to exceedance correlation, the tail dependence coefficient is generally easier to compute and analytical expressions can be obtained for almost all distributions. This is not the case for exceedance correlation even for usual distributions. Moreover, we can easily derive the tail dependence of a mixture from the tail dependence of the different components of the mixture. The last property will be used below to investigate which model can or cannot reproduce the results of Longin and Solnik (2001).

2.3. Why classical multivariate GARCH and RS model cannot reproduce asymptotic asymmetries

Ang and Chen (2002) and Ang and Bekaert (2002) try to reproduce asymmetric correlations facts with classical models such as GARCH and RS based on a multivariate normal distribution. After examining a number of models, they found that GARCH with constant correlation and fairly asymmetric GARCH cannot reproduce the asymmetric correlations documented by Longin and Solnik. However, they found that a RS model with Gaussian innovations is better at reproducing asymmetries in exceedance correlation. They clearly reproduce asymmetric correlations at finite distance. However, their finite-distance asymmetric correlation comes from the asymmetries produced in the marginal distributions with a regime switching in means, as suggested by the simulation in the previous section.[12] Therefore it becomes difficult to distinguish asymmetries in dependence from asymmetry in marginal distributions. This is a problem of practical relevance since most return series exhibit asymmetry in volatility.

By reinterpreting Longin and Solnik (2001) results in term of TDC instead of asymptotic exceedance correlation, we show analytically that all these models cannot reproduce asymptotic asymmetry even if some can reproduce finite-distance asymmetry. These results are extended to the rejection of more general classes of return-generating processes. The key point of this result is the fact that many classes of models including Gaussian (or Student) GARCH and RS can be seen as mixtures of symmetric distributions. We establish the following result.

Proposition 2.1

(i) *Any GARCH model with constant mean and symmetric conditional distribution has a symmetric unconditional distribution and hence a symmetric TDC.*
(ii) *If the conditional distribution of a RS model has a zero TDC, then the unconditional distribution also has a zero TDC.*
(iii) *From a multivariate distribution with symmetric TDC, it is impossible to construct an asymmetric TDC with a mixture procedure (as GARCH, RS or any other) by keeping all marginal distributions unchanged across mixture components.*

Proof. See Appendix A.

This proposition allows us to argue that the classical GARCH or RS models cannot reproduce asymmetries in asymptotic tail dependence. Therefore, the classical GARCH models (BEKK, CCC or DCC) with constant mean can be seen as a mixture of symmetric distributions with the same first moments and therefore exhibit a symmetric tail dependence function as well as a symmetric TDC.[13] When the mean becomes time-varying as in the GARCH-M model the

[9] Longin and Solnik (2001) determine by simulation an optimal threshold and use the subsample beyond this threshold to estimate the asymptotic exceedance correlation. However, this shortcoming does not compromise the results of Longin and Solnik (2001) since they choose different levels of threshold and still obtain the same result.

[10] When tail dependence is estimated non-parametrically as in Caillault and Guegan (2005), the same problem of limited extreme data points occurs.

[11] For the logistic function with parameter α, the correlation coefficient of extremes is $1-\alpha^2$ (see Longin and Solnik, 2001). We find that the upper tail dependence coefficient is $2-2^{\alpha}$. Then, both coefficients are zero when α equals 1 and different from zero when α is different from 1.

[12] Ang and Bekaert (2002) note that the ability of a RS model (compared to a GARCH model) to reproduce asymmetries comes from the fact that it accounts for the persistence in both first and second moments. The GARCH model accounts for this persistence only in second moments. We provide analytical arguments to support this intuition.

[13] The BEKK proposed by Engle and Kroner (1995) is a straightforward generalization of the GARCH model to a multivariate case which guarantees positive definiteness of the conditional variance–covariance matrix. In the CCC model proposed by Bollerslev (1990) the correlation matrix is assumed to be constant, while in the DCC of Engle (2002) this matrix is dynamic.

unconditional distribution can allow asymmetry in correlation (Ang and Chen, 2002), but this asymmetry comes from the mixture of the marginal distributions. The resulting skewness cannot be completely disentangled from the asymmetric correlation, since correlations are affected by marginal changes. Similarly, the classical RS model with Gaussian innovations is a discrete mixture of normal distributions which has a TDC equal to zero on both sides. Therefore, by (ii) we argue that both its TDCs are zero. However, at finite distance, when the mean changes with regimes, the exceedance correlation is not symmetric. This asymmetry is found by Ang and Chen (2002) and Ang and Bekaert (2002) in their RS model, but it disappears asymptotically and it comes from the asymmetry created in the marginal distributions by regime switching in means. Hence, the asymmetries in correlation are not separable from the marginal asymmetry, exactly like in the GARCH-M case. The part (iii) of Proposition 2.1 extends this intuition in terms of more general multivariate mixture models based on symmetric innovations. Actually when the marginal distributions are the same across all symmetric TDC components of a mixture, it is impossible to create asymmetry in TDCs.

Two relevant issues arise from the above discussion. First, how can we separate the marginal asymmetries from the asymmetry in dependence? Second, how can we account not only for asymmetries at finite distance but also for asymptotic dependence? In the next section, we propose a flexible model based on copulas that addresses these two issues.

3. A copula model for asymmetric dependence

Our model aims at capturing the type of asymmetric dependence found in international equity markets. Our discussion in the last section showed that it is important to disentangle the marginal distributions from the dependence structure. Therefore, we need to allow for asymmetry in tail dependence, regardless of the possible marginal asymmetry or skewness. Copulas, also known as dependence functions, are an adequate tool to achieve this aim.

3.1. Disentangling the marginal distributions from dependence with copulas

Estimation of multivariate models is difficult because of the large number of parameters involved. Multivariate GARCH models are a good example since the estimation becomes intractable when the number of series being modeled is high. The CCC of Bollerslev (1990), the DCC of Engle (2002), and the RSDC of Pelletier (2006) deal with this problem by separating the variance–covariance matrix into two parts, one for the univariate variances of the different marginal distributions, the other for the correlation coefficients. This separation allows them to estimate the model in two steps. In the first step, they estimate the marginal parameters and use them in the estimation of the correlation parameters in a second step. Copulas offer a tool to generalize this separation while extending the linear concept of correlation to nonlinear dependence.

Copulas are functions that build multivariate distribution functions from their unidimensional margins. Let $X \equiv (X_1, \ldots, X_n)$ be a vector of n univariate variables. Denoting F the joint n-dimensional distribution function and $F_1, \ldots, F_n$ the respective margins of $X_1, \ldots, X_n$. Then the Sklar theorem states that there exists a function C called copula which joins F to $F_1, \ldots, F_n$ as follows[14]:

$$F(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n)) \tag{3.1}$$

This relation can be expressed in term of densities by differentiating with respect to all arguments. We can therefore write (3.1) equivalently as

$$f(x_1, \ldots, x_n) = c(F_1(x_1), \ldots, F_n(x_n)) \times \prod_{i=1}^{n} f_i(x_i) \tag{3.2}$$

where f represents the joint density function of the n-dimensional variable X and f_i the density function of the variable X_i for $i = 1, \ldots, n$. The copula density function is naturally defined by $c(u_1, \ldots, u_n) \equiv \frac{\partial^n}{\partial u_1, \ldots, \partial u_n} C(u_1, \ldots, u_n)$. Writing the joint distribution density in the above form, we understand why it can be said that a copula contains all information about the dependence structure.[15]

We now suppose that our joint distribution function is parametric and we separate the marginal parameters from the copula parameters. So the relation (3.2) can be expressed as:

$$f(x_1, \ldots, x_n; \delta, \theta) = c(u_1, \ldots, u_n; \theta) \times \prod_{i=1}^{n} f_i(x_i; \delta_i); \tag{3.3}$$
$$u_i = F_i(x_i; \delta_i) \quad \text{for } i = 1, \ldots, n$$

where $\delta = (\delta_1, \ldots, \delta_n)$ are the parameters of the different margins and θ denotes the vector of all parameters that describe dependence through the copula. Therefore, copulas offer a way to separate margins from the dependence structure and to build more flexible multivariate distributions.

More recent works allow some dynamics in dependence. In a bivariate context, Rodriguez (2007) introduces regime switching in both the parameters of marginal distributions and the copula function.[16] Jondeau and Rockinger (2006) capture the time-varying volatilities of the individual equity index return series by a GARCH model and introduce Markov-switching Student-*t* copulas for pairs of countries. Recently, Chollete et al. (2009) propose a model of multivariate regime-switching copulas to capture asymmetric dependence in international financial returns. Ang and Bekaert (2002, 2004) allow all parameters of the multivariate normal distribution to change with the regime. The extension of these models to a large number of series faces the above-mentioned curse of dimensionality. Since the switching variable is present in both the margins and the dependence function, separation of the likelihood function into two parts is not possible and the two-step estimation cannot be performed. Pelletier (2006) uses the same separation as in the CCC or DCC and introduces the regime switching variable only in the correlation coefficients. By doing so, he can proceed with the two-step procedure to estimate the model while limiting the number of parameters to be estimated.[17] We carry out a similar idea but for nonlinear dependence.

Therefore, we separate the modeling of marginal distributions from the modeling of dependence by using univariate GARCH models for the marginal distributions and introducing changes in regime in the copula dependence structure. The pattern of the model with four variables (two countries, two markets in our following application) is illustrated in Fig. 3. The four marginal distributions are linked through a dependence function with two regimes, one symmetric, the other asymmetric.

[14] See Nelsen (1999) for a general presentation. Note that if F_i is continuous for any $i = 1, \ldots, n$ then the copula C is unique.

[15] The tail dependence coefficients are easily defined through a copula as $\tau^L = \lim_{\alpha \to 0} \frac{C(\alpha,\alpha)}{\alpha}$ and $\tau^U = \lim_{\alpha \to 0} \frac{2\alpha - 1 + C(1-\alpha, 1-\alpha)}{\alpha}$.

[16] The models proposed by Rodriguez (2007) in his analysis of contagion can reproduce asymmetric dependence but it cannot distinguish between skewness and asymmetry in the dependence structure. In fact, a change in regime produces both skewness and asymmetric dependence, two different features that must be characterized separately. The analysis is limited to pairs of stock markets in Asia and Latin America.

[17] Since Pelletier (2006) uses the normal distribution with constant mean, the resulting unconditional distribution is symmetric and cannot reproduce asymmetric dependence.

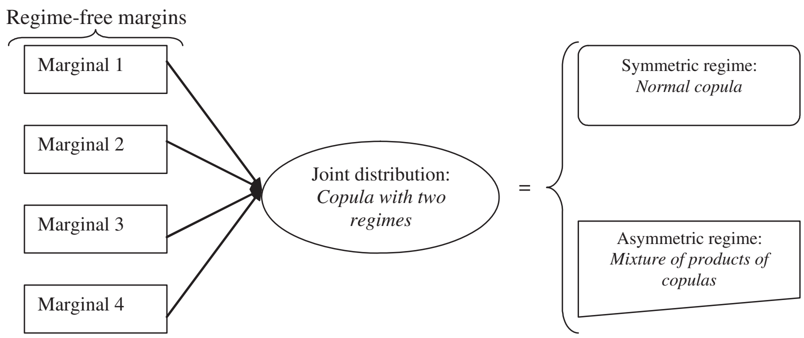

Fig. 3. Model structure: disentangling marginal distributions from the dependence structure with a two-regime copula, with one symmetric regime and one asymmetric regime. The marginal distributions are regime-free.

3.2. Specification of the marginal distributions

For marginal distributions, we use a M-GARCH (1, 1) model similar to Heston and Nandi (2000):

$$x_{i,t} = \mu_i + \lambda_i \sigma_{i,t}^2 + \sigma_{i,t} z_{i,t}; \quad z_{i,t} \sim N(0,1); \quad i = 1, \dots, 4 \tag{3.4}$$

$$\sigma_{i,t}^2 = \omega_i + \beta_i \sigma_{i,t-1}^2 + \alpha_i \left(z_{i,t-1} - \gamma_i \sigma_{i,t-1} \right)^2 \tag{3.5}$$

The variables $x_{1,t}$ and $x_{2,t}$ represent the log returns of equities and bonds respectively for the first country while $x_{3,t}$ and $x_{4,t}$ are the corresponding series for the second country; $\sigma_{i,t}^2$ denotes the conditional variance of $x_{i,t}$, λ_i can be interpreted as the price of risk and γ_i captures potential asymmetries in the volatility effect.[18] In the Heston and Nandi (2000) interpretation, μ_i represents the interest rate.[19] The parameters of the marginal distributions are grouped into one vector $\delta \equiv (\delta_1, \dots, \delta_4)$, with $\delta_i = (\mu_i, \lambda_i, \omega_i, \beta_i, \alpha_i, \gamma_i)$.

3.3. Specification of the dependence structure

Our dependence model is characterized by two regimes, one Gaussian regime in which dependence is symmetric (C_N) and a second regime that can capture the asymmetry in extreme dependence (C_A). The conditional copula is given by:

$$\begin{aligned} C(u_{1,t}, \dots, u_{4,t}; \rho^N, \rho^A | s_t) = {} & s_t C_N(u_{1,t}, \dots, u_{4,t}; \rho^N) \\ & + (1 - s_t) C_A(u_{1,t}, \dots, u_{4,t}; \rho^A) \end{aligned} \tag{3.6}$$

where $u_{i,t} = F_{it}(x_{i,t}; \delta_i)$, with F_{it} denoting the conditional cumulative distribution function of $x_{i,t}$ given the past observations. The variable s_t follows a Markov chain with a constant transitional probability matrix.

$$\begin{aligned} & M = \begin{pmatrix} P & 1-P \\ 1-Q & Q \end{pmatrix}; P = \Pr(s_t = 1 | s_{t-1} = 1) \text{ and} \\ & Q = \Pr(s_t = 0 | s_{t-1} = 0) \end{aligned} \tag{3.7}$$

The normal regime ($s_t = 1$) corresponds to the symmetric regime where the conditional joint normality can be supported and the asymmetric regime ($s_t = 0$) corresponds to the asymmetric regime in which markets are strongly more dependent for large negative returns than for large positive returns.

The Gaussian copula C_N is defined straightforwardly by (3.1) where the joint distribution $F = \Phi_{\rho^N}$ is the four-dimensional normal cumulative distribution function with all diagonal elements of the covariance matrix equal to one, i.e. $C_N(u_1, \dots, u_4; \rho^N) = \Phi_{\rho^N}(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_4))$, where Φ is the univariate standard normal cumulative distribution function.

The asymmetric components of the copula are illustrated in Fig. 4. The first one is characterized by independence between the two countries, but possibly extreme dependence between equities and bonds for each country. The second one is characterized by independence between equity and bond markets but allows for extreme dependence between equity returns and bond returns separately. The third one allows for possible extreme dependence between bonds in one country and equities in another country but supposes independence for the rest.

Formally, the asymmetric copula is the mixture of these three components and is expressed as follows:

$$\begin{aligned} C_A(u_1, \dots, u_4; \rho^A) \equiv {} & \pi_1 C_{GS}(u_1, u_2; \tau_1^L) \times C_{GS}(u_3, u_4; \tau_2^L) \\ & + \pi_2 C_{GS}(u_1, u_3; \tau_3^L) \times C_{GS}(u_2, u_4; \tau_4^L) \\ & + (1 - \pi_1 - \pi_2) C_{GS}(u_1, u_4; \tau_5^L) \times C_{GS}(u_2, u_3; \tau_6^L) \end{aligned} \tag{3.8}$$

with $\rho^A = (\pi_1,\ \pi_2,\ \tau_1^L,\ \tau_2^L,\ \tau_3^L,\ \tau_4^L,\ \tau_5^L,\ \tau_6^L)$, and the bivariate component is the Gumbel survival copula given by

$$\begin{aligned} C_{GS}(u, v; \tau^L) = {} & u + v - 1 \\ & + \exp\left[-\left((-\log(1-u))^{\theta(\tau^L)} + (-\log(1-v))^{\theta(\tau^L)} \right)^{1/\theta(\tau^L)} \right] \end{aligned} \tag{3.9}$$

where $\theta(\tau^L) = \frac{\log(2)}{\log(2-\tau^L)}$, $\tau^L \in [0, 1)$ is the lower TDC and the upper TDC is zero. As illustrated by Fig. 1, the choice of this particular copula is justified by its ability to replicate the pattern of dependence observed in the data. The shape of the exceedance correlation for the rotated Gumbel copula is more similar to the asymmetric shape observed in the data, while as expected Gaussian or RS with Gaussian distributions fail to replicate this shape.[20]

One can notice that our asymmetric copula specification implies some restrictions in the dependence structure. For three different couples from different components of this copula, the sum of their TDC is lower than one. Without any restrictions this sum may reach 3. For example, the TDC between bonds and equities in the first country is $\pi_1 \tau_1^L$, between equities of two countries $\pi_2 \tau_3^L$, and between equities in the first country and bonds in the second country $(1 - \pi_1 - \pi_2)\tau_5^L$. Therefore, the sum is $\pi_1 \tau_1^L + \pi_2 \tau_3^L + (1 - \pi_1 - \pi_2)\tau_5^L \leqslant 1$, since $\tau_1^L \leqslant 1$, $\tau_3^L \leqslant 1$, and $\tau_5^L \leqslant 1$. We explore the empirical implications of these restrictions in Section 4.4. A major problem in building multivariate distributions is how to

[18] The condition $\beta_i + \alpha_i \gamma_i^2 < 1$ is sufficient to have the stationarity of the process $x_{i,t}$ with finite unconditional mean and variance (see Heston and Nandi, 2000).

[19] Here we keep μ_i as a free parameter to give more flexibility to our model.

[20] The Longin and Solnik (2001) result implies that lower tails are dependent while upper tails are independent. Hence, the rotated Gumbel copula is designed to model this feature since it has this tail dependence structure.

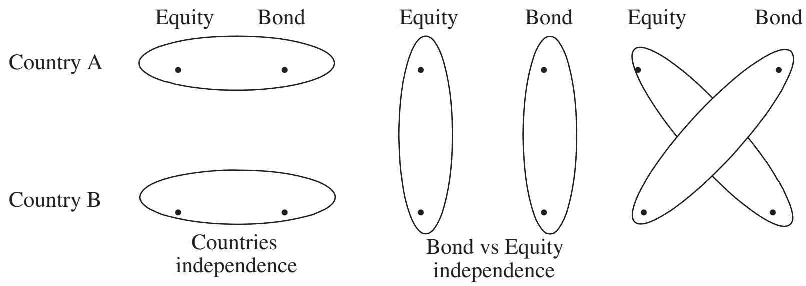

Fig. 4. Illustration of the three components of asymmetric copula. Each component is the product of the two bivariate copulas representing the corresponding encircled couple of returns.

construct multivariate copulas with specific bivariate marginal distributions.[21] A theorem by Genest et al. (1995) states that it is not always possible to construct multivariate copulas with given bivariate margins. Therefore, even if in the bivariate case we can have a nice asymmetric copula with lower tail dependence and upper tail independence as Longin and Solnik (2001) suggest, some problems remain when we contemplate more than two series. Most existing asymmetric tail-dependent copulas are in the family of Archimedean copulas and the usual straightforward generalization in multivariate copulas constrains all bivariate marginal copulas to be the same. This is clearly not admissible in the context of our analysis. In the above model, we allow each of the six couples of interest to have different levels of lower TDC. As C_A is constructed, it is easy to check that it is a copula since each component of the mixture is a copula and the mixture of copulas is a copula.[22]

It is important to notice that, in this model, the labeling of each regime is defined ex-ante. The normal regime ($s_t = 1$) corresponds to the symmetric regime where the conditional joint normality can be supported and the asymmetric regime ($s_t = 0$) corresponds to the asymmetric regime in which markets are strongly more dependent for large negative returns than for large positive returns.

3.4. An adapted parsimonious model

Given our application, we impose an additional constraint: $\pi_1 + \pi_2 = 1$. This means that we neglect the asymmetric cross-dependence between equities in one country and bonds in another country. However, it should be stressed that we maintain cross-country dependence through the normal regime. In Section 4.4, we investigate the effect of this constraint on the overall the level of dependence measured by the Spearman rho, a nonparametric measure of dependence.[23] The mixed copula becomes:

$$\begin{aligned} C_A(u_1,\ldots,u_4;\rho^A) \equiv\ & \pi C_{GS}(u_1,u_2;\tau_1^L)\times C_{GS}(u_3,u_4;\tau_2^L) \\ & +(1-\pi)C_{GS}(u_1,u_3;\tau_3^L)\times C_{GS}(u_2,u_4;\tau_4^L) \end{aligned} \quad (3.10)$$

Therefore, the asymmetric copula is now characterized by just five parameters $\rho^A = (\pi,\ \tau_1^L,\ \tau_2^L,\ \tau_3^L,\ \tau_4^L)$.

3.5. Estimation

As already mentioned, our structure allows for a two-step estimation procedure. The likelihood function must be evaluated unconditionally to the unobservable regime variable s_t and decomposed in two parts. Let us denote the sample of observed data by $\underline{X}_T = \{X_1,\ldots,X_T\}$ where $X_t \equiv \{x_{1,t},\ldots,x_{4,t}\}$. The log likelihood function is given by:

$$L(\delta,\theta;\underline{X}_T) = \sum_{t=1}^{T} \log f(X_t;\delta,\theta|\underline{X}_{t-1}) \quad (3.11)$$

where $\underline{X}_{t-1} = \{X_1,\ldots,X_{t-1}\}$ and θ is a vector including the parameters of the copula and the transition matrix. Hamilton (1989) describes a procedure to perform this type of evaluation[24]. With $\xi_t = (s_t, 1-s_t)'$ and denoting

$$\eta_t = \begin{bmatrix} f(X_t;\delta,\theta|\underline{X}_{t-1}, s_t = 1) \\ f(X_t;\delta,\theta|\underline{X}_{t-1}, s_t = 0) \end{bmatrix} \quad (3.12)$$

the density function conditionally to the regime variable s_t and the past returns can be written as:

$$f(X_t;\delta,\theta|\underline{X}_{t-1}, s_t) = \xi_t'\eta_t \quad (3.13)$$

Since s_t (or ξ_t) is unobservable, we integrate on s_t and obtain the unconditional density function:

$$\begin{aligned} f(X_t;\delta,\theta|\underline{X}_{t-1}) = & \Pr[s_t = 1|\underline{X}_{t-1};\delta,\theta]\times f(X_t;\delta,\theta|\underline{X}_{t-1}, s_t = 1) \\ & +\Pr[s_t = 0|\underline{X}_{t-1};\delta,\theta]\times f(X_t;\delta,\theta|\underline{X}_{t-1}, s_t = 0) \end{aligned} \quad (3.14)$$

The conditional probabilities of being in different regimes at time t conditional on observations up to time $t-1$, denoted by $\hat{\xi}_{t|t-1} \equiv (\Pr[s_t = 1|\underline{X}_{t-1};\delta,\theta],\ \Pr[s_t = 0|\underline{X}_{t-1};\delta,\theta])'$, are computed through the Hamilton filter. Starting with the initial value $\hat{\xi}_{1|0}$, the optimal inference and forecast for each date in the sample is given by the iterative equations:

$$\hat{\xi}_{t/t} = \left[\hat{\xi}_{t|t-1}'\eta_t\right]^{-1}\left(\hat{\xi}_{t|t-1}\odot\eta_t\right) \quad (3.15)$$

$$\hat{\xi}_{t+1/t} = M'\cdot\hat{\xi}_{t|t} \quad (3.16)$$

where $\odot$ denotes element-by-element multiplication. Finally, the unconditional density can be evaluated with the observed data as $f(X_t;\delta,\theta|\underline{X}_{t-1}) = \hat{\xi}_{t|t-1}'\eta_t$ and the log likelihood becomes:

$$L(\delta,\theta;\underline{X}_T) = \sum_{t=1}^{T}\log\left(\hat{\xi}_{t|t-1}'\eta_t\right) \quad (3.17)$$

[21] Nelsen (1999, p. 86) mentions that it may be the most important open question concerning copulas. Aas et al. (2007) propose an approach to build multiple dependence based on pair-copula decomposition. Their approach proceeds by a hierarchical incorporation of more variables in the conditioning sets. This procedure provides a nice way to build flexible multivariate copula. In practice it is important to make a good choice of couples that should be used in the first level of the hierarchy since a limitation of the procedure is that the couples after the first level of the hierarchy are based on conditional copulas.

[22] A copula can be seen as the cdf of a multidimensional variable with uniform [0,1] margins. If we consider two bivariate independent variables with uniform margins the copula linking the four variables is simply the product of the corresponding bivariate copulas. Hence, such a product is always a copula.

[23] A related study by Hartmann et al. (2004) using extreme value theory tends to support this restriction. Analyzing stock and bond returns for G-5 countries, they find that extreme dependence between stocks and bonds is much lower that extreme dependence between stock markets or bond markets. This is especially the case for cross-country dependence between stocks in one country and bonds in another country.

[24] A general presentation can be found in Hamilton (1994, chapter 22).

To perform the two-step procedure, we decompose the log likelihood function into two parts: the first part includes the likelihood functions of all margins, while the second part represents the likelihood function of the copula.

Proposition 3.2 (*Decomposition of the log likelihood function*). *The log likelihood function can be decomposed into two parts including the margins and the copula*

$$L(\delta,\theta;\underline{X}_T)=\sum_{i=1}^{4}L_i\left(\delta_i;\ \underline{X}_{i,T}\right)+L_C(\delta,\ \theta;\ \underline{X}_T) \tag{3.18}$$

where

$$\underline{X}_{i,t}=\{x_{i,1},\ldots,x_{i,t}\}$$

$$L_i(\delta_i;\ \underline{X}_{i,T})=\sum_{t=1}^{T}\log f_i(x_{i,t};\ \delta_i|\underline{X}_{i,t-1})$$

$$L_C(\delta,\ \theta;\ X)=\sum_{t=1}^{T}\log\left(\hat{\xi}'_{t|t-1}\eta_{ct}\right)$$

with

$$\eta_{ct}=\begin{bmatrix}c(u_{1,t}(\delta_1),\ldots,u_{n,t}(\delta_n);\ \theta|s_t=1)\\ c(u_{1,t}(\delta_1),\ldots,u_{n,t}(\delta_n);\ \theta|s_t=0)\end{bmatrix};\quad u_{i,t}(\delta_i)=F_i\left(x_{i,t};\ \delta_i|\underline{X}_{i,t-1}\right)$$

and $\hat{\xi}'_{t|t-1}$ *filtered from* η_{ct} *as*

$$\hat{\xi}_{t/t}=\left[\hat{\xi}'_{t|t-1}\eta_{ct}\right]^{-1}(\hat{\xi}_{t|t-1}\odot\eta_{ct})$$

$$\hat{\xi}_{t+1/t}=M'\cdot\hat{\xi}_{t|t}$$

Proof. See Appendix A.

Several options are available for the estimation of the initial value $\hat{\xi}_{1|0}$. One approach is to set it equal to the vector of unconditional probabilities, which is the stationary transitional probability of the Markov chain. Another simple option is to set $\hat{\xi}_{1|0}=N^{-1}1_N$. Alternatively it could be considered as another parameter, which will be estimated subject to the constraint that $1'_N\hat{\xi}_{1|0}=1$. We will use the first option here.

Through the above decomposition, we notice that each marginal log likelihood function is separable from the others. Therefore, even if the estimation of all margins is performed in a first step, we can estimate each set of marginal parameters separately into this step. The first step is then equivalent to n single estimations of univariate distributions. The two-step estimation is formally written as follows:

$$\hat{\delta}=\underset{\delta=(\delta_1,\ldots,\delta_4)\in\Delta}{\operatorname{argmax}}\sum_{i=1}^{4}L_i\left(\delta_i;X_{i,.}\right), \tag{3.19}$$

$$\hat{\theta}=\underset{\theta\in\Theta}{\operatorname{argmax}}\,L_C(\hat{\delta},\theta;X). \tag{3.20}$$

The estimator for the parameters of the marginal distributions is then $\hat{\delta}=(\hat{\delta}_1,\ldots,\hat{\delta}_4)$, with $\hat{\delta}_i=(\hat{\mu}_i,\hat{\lambda}_i,\hat{\omega}_i,\hat{\beta}_i,\hat{\alpha}_i,\hat{\gamma}_i)'$; and $\hat{\theta}=(\hat{\rho}^N;\hat{\rho}^A;\widehat{P};\widehat{Q})$ includes all estimators of the parameters involved in the dependence structure. Δ and Θ represent the sets of all possible values of δ and θ respectively.

3.6. Tests of dependence structure

We perform three different tests for dependence structure. The first one is the Longin and Solnik (2001) test for exceedence correlation. For this test, we focus on the lower tail dependence to compare exceedence correlation of return data, the simulated return from two RS with Gaussian distributions, and the simulated return from a Gumbel copula model.

We also test the asymmetry in dependence. While the proposed copula model has the potential to capture asymmetry in dependence, we need to test formally for the presence of such asymmetric dependence. The natural way to evaluate whether dependence is asymmetric is to test the null hypothesis of one normal copula regime (H_0: (P = 1 and Q = 0), where P and Q are the parameters of the transition probability matrix), against the alternative hypothesis of two-copula regimes including the normal one and the asymmetric one. This test faces the general problems found in testing in RS models. In particular, under the null hypothesis, nuisance parameters are unidentified and the scores are identically zero.[25]

Maximized Monte Carlo (MMC) tests of Dufour (2006), which are a generalization of classical Monte Carlo (MC) tests of Dwass (1957) and Barnard (1963), are adapted for tests facing these problems. The MC tests of Dwass (1957) and Barnard (1963) are performed by doing many replications (with the same sample size as the data sample) under the null hypothesis, and compute the test statistic for each replication. The distribution of the test statistic is therefore approximated by the distribution of the obtained values. One can therefore compute the value of the test statistic with the data and deduce from the MC distribution the p-value of the test. The classical MC test does not deal with the presence of nuisance parameters under the null hypothesis. The MMC of Dufour (2006) addresses the problem of nuisance parameters under the null. When the test statistic involves the nuisance parameters as in the case of the likelihood ratio test under the alternative, the values of these parameters are needed to compute the test statistic on simulated data.

The MMC technique is the maximization of the p-values given all the possible values of the nuisance parameters. This test is computationally very demanding. However, Dufour (2006) proposes a simplified version that focuses on the estimated values of the nuisance parameters and shows that it works under the assumptions of uniform continuity, and convergence over the nuisance parameter space. Our model satisfies these assumptions of uniform continuity and convergence. Therefore, we can apply this simpler version also known as parametric bootstrap test.

Finally, we compare the goodness-of-fit of this model with two different structures. Chen and Fan (2005) propose a test to compare non-nested copula models. We apply this test to compare our model with a regime switching model where both regimes display Gaussian copulas. We also compare our model with a model with just one regime (of Gumbel copula) which displays an asymmetric dependence behavior to make sure that it is necessary to use in addition, a symmetric regime (of Gaussian copula).

4. Dependence structure in international bond and equity markets: an empirical investigation

4.1. Data

We will consider the same model for two pairs of two countries. First, we model the equity and bond markets in the United States and Canada. The US equity returns are based on the SP 500 index, while the Canadian equity returns are computed with the Datastream index. The bond series are indices of 5-year government bonds computed by Datastream. These bond indices are available

[25] Hansen (1996) describes the asymptotic distributions of standard test statistics in the context of regression models with additive nonlinearity. Garcia (1998) and Hansen (1992) provide the asymptotic null distribution of the likelihood ratio test. Andrews and Ploberger (1994) address the first problem in a general context and derive an optimal test. The above procedures solve the problem of unidentified nuisance parameters under the null and the identically zero scores. However, there is an additional problem of testing parameter on the boundary. Andrews (2001) deals with this boundary problem but in the absence of the first two problems.

Table 1
Summary statistics of weekly bond and equity index returns for the four countries. All returns are in US dollars, from January 01, 1985 to December 21, 2004, which corresponds to a sample of 1044 observations. Sharpe ratio represents the ratio of the mean over the standard deviation of return.

	Mean[a]	Std[a]	Kurtosis	Skewness	Min[a]	Max[a]	Sharpe ratio
US equity	13.67	17.51	17.00	−1.55	−680.36	311.10	0.78
US bond	7.57	4.69	0.67	−0.06	−66.91	58.81	1.61
CA equity	11.24	16.72	13.62	−1.67	−610.87	225.15	0.67
CA bond	8.81	8.15	1.13	−0.24	−130.55	118.07	1.08
FR equity	14.72	23.43	7.18	−0.09	−582.12	512.16	0.63
FR bond	11.52	11.16	0.92	0.04	−142.02	166.68	1.03
DE equity	12.57	24.97	8.01	−0.46	−574.96	463.08	0.50
DE bond	10.44	11.56	0.82	−0.01	−142.54	171.39	0.90

[a] Annualized percent.

Table 2
Unconditional correlations between bonds and equity for US, Canada (CA), France (FR), and Germany (DE).

	US equity	US bond	CA equity	CA bond	FR equity	FR bond	DE equity
US bond	0.0576						
CA equity	0.7182	0.0116					
CA bond	0.1783	0.4706	0.4392				
FR equity	0.1957	−0.0182	0.1974	0.1065			
FR bond	−0.0499	0.3386	−0.0080	0.2433	0.3066		
DE equity	0.2089	−0.0536	0.1995	0.1009	0.8099	0.2625	
DE bond	−0.0832	0.3081	−0.0234	0.2143	0.3084	0.9403	0.2847

daily and are chain-linked allowing the addition and removal of bonds without affecting the value of the index.

We also consider France and Germany as a pair of countries. An additional interest here will be to see how the introduction of the European common currency changed the dependence structure between the asset markets in these two countries. The bond indices are the Datastream 5-year government bond indices, while the equity indices are the MSCI series.

All returns are total returns and are expressed in US dollars on a weekly basis from January 01, 1985 to December 21, 2004, which corresponds to a sample of 1044 observations. Descriptive statistics are reported in Table 1.

Sharpe ratios appear to be of the same magnitude for both equities and bonds, around 0.6 on average for the first and slightly above 1 for the second. The United States exhibits the highest ratios among the four countries. All return series present negative skewness except for the French bond index. Both mean returns and return volatility are higher in France and Germany than in the US and Canada. The volatility of returns in France and Germany is more than 23%, while it is only 18% in the US and Canada.

Unconditional correlations are reported in Table 2. The US and Canadian markets exhibit relatively high correlations, 0.72 for equities and 0.5 for bonds. The same is true for the France–Germany pair, although the bond markets are tightly linked, with a correlation of 0.94. The North-American equity markets are less correlated with European equity markets (around 0.2) than their bond counterparts (around 0.32). The cross-correlations between equity and bond markets vary from country to country. On average the two markets seem to move independently in the United States, while they are more closely related in Canada (0.44) and in Europe (around 0.3 for both France and Germany). Cross-correlations between equities and bonds in two different countries are not very high for US and Canada, and of the same order of magnitude than within-country cross-correlations (0.3) for France and Germany.

4.2. Marginal distributions

The estimates of the marginal parameters are reported in Table 3. The large values for the β_i parameters (around 90%) capture the high persistence in volatility. The high degree of significance for the parameter λ indicates that asset returns are skewed.

One important assumption for these GARCH models is that the error terms are i.i.d. Therefore, to verify if the assumption is fulfilled, we perform some tests of independence and normality on the residuals. The test results in Table 4 suggest that the independence assumption of residuals cannot be rejected for all series with a good degree of confidence.

4.3. Dependence structure in bond and equity markets

Three main conclusions emerge from the empirical results. First, there appears to be a large extreme cross-country dependence for both the equity and bond markets, while there is little dependence between equities and bonds in the same country. Second, the dependence structure exhibits a strong nonlinearity. Third, there seems to be a link between exchange rate volatility and asymmetry of dependence.

4.3.1. US–Canada dependence structure

In Table 5, we report the results of estimating the dependence model described in Section 3.4. The cross-country extreme dependence is large in both equity and bond markets, but the dependence across the two markets is relatively low in both countries. In the asymmetric regime, the TDCs are larger than 54% in both bond–bond and equity–equity markets, while both equity–bond TDCs in US and Canada are lower than 2%. This observation has an important implication for international diversification. The fact that extreme dependence in international equity and bond markets is larger than national bond–equity dependence can have a negative effect on the gain of international diversification and encourage the switching from equity to the domestic bond or risk-free asset in case of bear markets.

The average absolute value of correlation in the normal regime is larger than 39% for cross-country dependence and lower than 41% for equity–bond dependence. In the last case the correlation between bonds and equities in Canada is unusually high. The results underline the differences between unconditional correlation and the correlation in the normal regime. In fact, the presence of extreme

Table 3
Estimates of M-GARCH (1,1) parameters for all bond and equity returns of four countries. The figures between brackets represent standard deviations of the parameters. L is the value of the log likelihood function.

	US		CA		FR		DE	
	Equity	Bond	Equity	Bond	Equity	Bond	Equity	Bond
β	7.94e−1	7.82e−1	8.09e−1	9.07e−1	9.68e−1	9.36e−1	9.24e−1	9.56e−1
	(3.49e−1)	(1.62e−1)	(4.06e−1)	(1.79e−1)	(3.61e−1)	(4.21e−1)	(1.54e−1)	(2.45e−1)
α	5.46e−5	2.63e−6	6.40e−5	7.30e−6	2.28e−5	1.51e−5	2.22e−5	1.08e−5
	(4.04e−5)	(6.36e−5)	(8.16e−5)	(2.94e−5)	(9.35e−6)	(2.17e−5)	(2.14e−4)	(1.88e−5)
γ	4.45e+1	3.84e+1	2.73e+1	3.28e+1	1.91e+1	6.53e+0	1.19e+1	3.26e+0
	(1.70e−2)	(6.11e−3)	(1.14e−2)	(1.22e−2)	(1.61e−2)	(1.85e−1)	(8.07e−2)	(2.45e−2)
λ	1.72e+0	1.37e+1	3.13e+0	1.01e+1	1.61e+0	5.61e+0	1.78e+0	6.13e+0
	(1.39e−2)	(1.05e−2)	(2.09e−2)	(7.59e−3)	(7.22e−3)	(1.96e−1)	(6.33e−2)	(7.86e−3)
ω	7.57e−6	6.49e−6	1.21e−5	3.49e−6	1.99e−6	1.51e−7	6.46e−5	4.79e−7
	(9.64e−5)	(1.90e−5)	(1.74e−5)	(2.52e−5)	(6.53e−5)	(4.33e−5)	(1.92e−4)	(3.25e−5)
μ	1.07e−3	7.18e−4	1.32e−3	4.73e−4	1.48e−3	5.37e−4	6.51e−4	1.35e−4
	(1.29e−4)	(6.74e−5)	(3.76e−5)	(5.26e−5)	(5.00e−4)	(1.45e−4)	(1.32e−4)	(3.38e−5)
L	2.49e+3	3.77e+3	2.50e+3	3.20e+3	2.10e+3	2.88e+3	2.04e+3	2.84e+3

Table 4
Test for iid normal assumption of residuals of the M-GARCH models for the marginal distributions. The panel A provides the Box–Pierce and the Ljung–Box statistics for tests of independence of residuals of the M-GARCH models for the marginal distributions. For each series, the statistic is computed for different numbers of lags (1, 4, 6, and 12). The panel B presents the Kolmogorov–Smirnov (KS) and the Anderson–Darling (AD) statistics estimates with their p-values. A p-value greater than a given level (1, 5, or 10%) means that the normality assumption cannot be rejected at that level.

	US		CA		FR		DE	
	Equity	Bond	Equity	Bond	Equity	Bond	Equity	Bond
Panel A: Test for serial independence								
Box–Pierce								
12 lags	23.26*	18.57**	14.42**	9.27**	10.93**	9.88**	8.64**	12.19**
6 lags	14.85*	12.19**	10.26**	7.17**	10.70**	5.06**	4.55**	8.85**
4 lags	8.73**	10.49*	9.02**	6.34**	7.00**	3.7099**	3.39**	6.36**
1 lag	5.36*	0.01**	3.71**	0.45**	6.11*	1.33**	3.18**	2.78**
Ljung–Box								
12 lags	23.43*	18.71**	14.51**	9.32**	10.98**	9.97**	8.71**	12.28**
6 lags	14.93*	12.25**	10.31**	7.20**	10.74**	5.09**	4.57**	8.90**
4 lags	8.76**	10.55*	9.05**	6.37**	7.02**	3.7248**	3.40**	6.38**
1 lag	5.37*	0.01**	3.72**	0.45**	6.13*	1.33**	3.19**	2.79**
Panel B: Normality test								
KS								
Statistics	0.0238	0.0196	0.0295	0.0241	0.0230	0.0252	0.0180	0.0203
p-Value	0.5926	0.8146	0.3204	0.5738	0.6334	0.5174	0.8855	0.7772
AD								
Statistics	0.6197	0.6536	1.3762	1.1604	0.7035	0.7075	0.6007	0.7707
p-Value	0.6295	0.5988	0.2090	0.2831	0.5559	0.5526	0.6473	0.5026

* We cannot reject independence at the 1% level.
** We cannot reject independence at the 5% level.

dependence in the negative returns explains this difference since the multivariate Gaussian distribution has independence in the tails of returns regardless of the level of correlation.

The separation of the distribution into two parts, including the normal regime and the asymmetric regime, allows to capture the strong nonlinear pattern in the dependence structure. Moreover, it is interesting to see that for a high unconditional correlated couple such as the US and Canada equity markets, this separation gives not only an extreme dependence for the asymmetric regime, but also a high correlation in the normal regime (87%) that appears larger than the unconditional correlation (72%). This result may seem counter-intuitive if we take the unconditional correlation as a "mean" of the correlations in the two regimes. Of course, one must realize that the asymmetric regime can be characterized by a low correlation but by a large TDC. This demonstrates the importance of distinguishing between correlation and extreme dependence. The mixture model is better able to capture this distinction in fitting the data. A normal distribution may be a good approximation for measuring finite distance dependence, but an appropriate copula structure is necessary for characterizing extreme dependence.

4.3.2. France–Germany dependence structure

The estimation results are shown in Table 6. Due to a high cross-country unconditional correlation in both markets, the results for France and Germany are more eloquent. The dependence between equities and bonds is low, while the dependence between assets of the same type is large in both regimes. For France and Germany, equity–equity correlation and bond–bond correlation are larger than 90% while bond–equity correlations are lower than 21% in the same country as well as between the two countries. In the asymmetric regime, the TDC are larger than 67% between assets

Table 5
Dependence structure between the United States and Canada in equity and bond markets. Correlation coefficients are reported for the normal regime, while tail dependence coefficients describe the asymmetric regime. The tail dependence coefficient is obtained as the product of parameter τ and the respective weight π for cross-asset dependence and $1-\pi$ for cross-country dependence. Standard deviations are reported between parentheses for all parameters estimated directly from the model. The last raw reports the diagonal elements of the transition probability matrix.

Cross-country (US–CA) dependence

	Normal regime Correlation coefficient	Asymmetric regime Tail dependence coefficient		
			τ	TDC $((1-\pi)\tau)$
US equity–CA equity	0.8739 (0.1560)		0.9100 (0.0185)	0.7917
US bond–CA bond	0.3870 (0.0831)		0.6234 (0.0124)	0.5424
		$1-\pi$	0.6897	

Cross-asset (equity–bond) dependence

	Normal regime Correlation coefficient		Asymmetric regime Tail dependence coefficient		
	US bond	CA bond		τ	TDC $(\pi\tau)$
US equity	−0.1101 (0.0416)	0.1234 (0.0312)	US equity–US bond	0.1300 (0.041)	0.0169
CA equity	−0.0812 (0.0207)	0.4085 (0.0103)	CA equity–CA bond	0.1385 (0.0145)	0.0180
			π	0.3102 (0.0207)	

Parameters of transitional probability matrix

P	0.9020 (0.0207)	Q	0.9586 (0.0206)

Table 6
Dependence structure between France and Germany in equity and bond markets. Correlation coefficients are reported for the normal regime, while tail dependence coefficients describe the asymmetric regime. The tail dependence coefficient is obtained as the product of parameter τ and the respective weight π for cross-asset dependence and $1-\pi$ for cross-country dependence. Standard deviations are reported between parentheses for all parameters estimated directly from the model. The last raw reports the diagonal elements of the transition probability matrix.

Cross-country (FR–DE) dependence

	Normal regime Correlation coefficient	Asymmetric regime Tail dependence coefficient		
			τ	TDC $((1-\pi)\tau)$
FR equity–DE equity	0.9083 (0.0267)		0.9554 (0.0603)	0.7787
FR bond–DE bond	0.9901 (0.058)		0.8261 (0.027)	0.6733
		$1-\pi$	0.8151	

Cross-asset (equity–bond) dependence

	Normal regime Correlation coefficient		Asymmetric regime Tail dependence coefficient		
	FR Bond	DE Bond		τ	TDC $(\pi\tau)$
FR equity	0.1893 (0.0170)	0.2023 (0.0129)	FR equity–FR bond	0.0923 (0.028)	0.0171
DE equity	0.1175 (0.0214)	0.1294 (0.030)	DE equity–DE bond	0.0969 (0.029)	0.0179
			π	0.1849 (0.0294)	

Parameters of transitional probability matrix

P	0.8381 (0.0270)	Q	0.9373 (0.0373)

of the same type and lower than 2% between bond and equities in both France and Germany.[26]

To analyze the effect of the Euro on the dependence structure, we split the observation period in two subperiods, before and after the introduction of the currency. Tables 7 and 8 contain the results for the respective subperiods. We find that the introduction of the Euro increases the correlation in the normal regime between the French and German markets. Before the introduction of the Euro, in the normal regime, the cross-country correlation between assets of the same type is on average 80%, against more than 96% after the introduction. The cross-asset correlations exhibit a similar pattern since all correlations increase after the introduction of the Euro. This result is consistent with those of Cappiello et al. (2006) who find that the introduction of a fixed exchange rate leads to a structural break characterized by a high correlation.[27] For the asymmetric regime, the results are more surprising since the extreme dependence between the French and German equity markets drastically decreases from 87% to 26%. All the other extreme dependence coefficients increase, but only the TDC of the FR bond–DE bond pair increases significantly. Since this change in the level of dependence suggests a relationship between the dependence structure and the exchange rate, we investigate it further in the next section for both pairs of countries.

These results for the two pairs of countries that we analyzed suggest that an asymmetric regime cannot be readily associated with market situations such as bull and bear markets or calm and volatile markets used in the previous literature on contagion. Das and Uppal (2004) relate high volatility and downturn markets to a large conditional correlation,[28] while Forbes and Rigobon (2002) define contagion as a change in correlation during more volatile market times. In an asymmetric regime of dependence, extreme negative shocks are more likely to be transmitted to other markets than positive shocks, irrespective of the market situation. In the normal dependence regime good and bad shocks are transmitted with the same probability.

4.3.3. Dependence structure tests

Empirical results for the Longin–Solnik tests (see Table 9) show that the RS model with one symmetric regime and one asymmetric regime displays significant exceedence correlations for the lower tail present in the data, while the classical RS model with Gaussian distributions fails to do so. The MMC of Dufour (2006) test confirms the presence of asymmetry in the dependence structure. Finally when using the Chen and Fan (2005) test to compare the RS with one Gaussian regime and one rotated Gumbel regime, the RS with two Gaussian copulas, and the model just with rotated Gumbel copula, we find that the RS model with one Gaussian copula and one rotated Gumbel copula outperforms the two other models.

4.3.4. Link between asymmetric dependence and the exchange rate

The filtered probabilities to be in asymmetric regime are obtained as a by-product of estimation. They provide at each time period t a probabilistic assessment of being in the asymmetric regime conditional on the information available at time t (Fig. 5). For

[26] We also estimated a version of the model where we allowed dependence between equities and bonds across Germany and France but not within each country. We found TDCs even lower than 2%, which tends to support our initial assumption to ignore such cross-country dependence across markets.

[27] The goal of Cappiello et al. (2006) was to investigate the asymmetric effect of past news on the correlation. Since it is well documented that the negative shocks have a larger effect on volatility than the positive shocks of the same magnitude, they try to see if the result is similar for correlation.

[28] Boyer et al. (1999) and Forbes and Rigobon (2002) show that there is a theoretical bias when comparing correlations in different market situations.

Table 7
Subperiod I (period before the introduction of the Euro currency: from January 01, 1985 to December 29, 1998 for a sample of 731 observations). Dependence structure between France and Germany in equity and bond markets. Correlation coefficients are reported for the normal regime, while tail dependence coefficients describe the asymmetric regime. The tail dependence coefficient is obtained as the product of parameter τ and the respective weight π for cross-asset dependence and $1-\pi$ for cross-country dependence. Standard deviations are reported between parentheses for all parameters estimated directly from the model. The last raw reports the diagonal elements of the transition probability matrix.

Cross-country (FR–DE) dependence				
Normal regime Correlation coefficient		Asymmetric regime Tail dependence coefficient		
			τ	TDC $((1-\pi)\tau)$
FR equity–DE equity	0.6924		0.9554	0.8663
	(0.0760)		(0.035)	
FR bond–DE bond	0.9082		0.8388	0.7606
	(0.038)		(0.061)	
		$1-\pi$	0.9067	

Cross-asset (equity–bond) dependence					
Normal regime Correlation coefficient			Asymmetric regime Tail dependence coefficient		
	FR bond	DE bond		τ	TDC $(\pi\tau)$
FR equity	0.2091	0.1641	FR equity–FR bond	0.1130	0.0105
	(0.0123)	(0.0151)		(0.021)	
DE equity	0.1205	0.1519	DE equity–DE bond	0.0067	0.0006
	(0.0106)	(0.049)		(0.072)	
			π	0.0933	
				(0.010)	

Parameters of transitional probability matrix			
P	0.0651	Q	0.9438
	(0.0103)		(0.0102)

Table 8
Subperiod II (period after the introduction of the Euro currency: from January 05, 1999 to December 21, 2004 for a sample of 313 observations). Dependence structure between France and Germany in equity and bond markets. Correlation coefficients are reported for the normal regime, while tail dependence coefficients describe the asymmetric regime. The tail dependence coefficient is obtained as the product of parameter τ and the respective weight π for cross-asset dependence and $1-\pi$ for cross-country dependence. Standard deviations are reported between parentheses for all parameters estimated directly from the model. The last raw reports the diagonal elements of the transition probability matrix.

Cross-country (FR–DE) dependence				
Normal regime Correlation coefficient		Asymmetric regime Tail dependence coefficient		
			τ	TDC $((1-\pi)\tau)$
FR equity–DE equity	0.9426		0.2598	0.2582
	(0.0950)		(0.0106)	
FR bond–DE Bond	0.9937		0.8946	0.8892
	(0.0382)		(0.071)	
		$1-\pi$	0.9940	

Cross-asset (equity–bond) dependence					
Normal regime Correlation coefficient			Asymmetric regime Tail dependence coefficient		
	FR bond	DE bond		τ	TDC $(\pi\tau)$
FR equity	0.2272	0.2350	FR equity–FR bond	0.2249	0.0013
	(0.0241)	(0.0177)		(0.024)	
DE equity	0.1516	0.1573	DE equity–DE bond	0.9760	0.0059
	(0.0118)	(0.059)		(0.082)	
			π	0.0060	
				(0.012)	

Parameters of transitional probability matrix			
P	0.9212	Q	0.2274
	(0.0118)		(0.0117)

France and Germany, these probabilities show a clear break after the introduction of the Euro. Before its introduction, the dependence is more likely asymmetric and becomes more Gaussian after the event. To investigate this relationship further, we perform a logistic regression of the conditional probabilities to be in the asymmetric regime on the volatility of the exchange rate.[29]

For France and Germany, we have:

$$\widehat{P}_t = \underset{\substack{-1.26e+0 \\ (6.81e-2)}}{a} + \underset{\substack{5.06e+2 \\ (2.29e+1)}}{b} \times Vol_t + e_t$$

The dependent variable is $\widehat{P}_t = \log(P_t/(1-P_t))$, where P_t is the conditional probability to be in the asymmetric regime given the time-t available information, and Vol_t is the exchange rate volatility between the two countries obtained by a M-GARCH (1,1) filter. Standard deviations are reported between parentheses. The coefficient is positive and highly significant (the R^2 of the regression is 0.86) suggesting a strong relationship between exchange rate volatility and asymmetric dependence.

One may be concerned that the relation between the probability of the asymmetric regime and exchange rate volatility is due to the correlation of the latter with the volatility of the equity market or the bond market of the respective countries. To address this issue, we perform an orthogonalization. We regress the exchange rate on all equity and bond return volatilities in a first step and keep residuals. Then we regress the probability of asymmetric regime on these residuals in a second step. The relation remains significant.

We run the same regression for US and Canada to investigate if the relation holds when no structural change occurs. The results are similar to the European results.

$$\widehat{P}_t = \underset{\substack{-7.71e-1 \\ (1.76e-1)}}{a} + \underset{\substack{9.30e+1 \\ (2.36e+1)}}{b} \times Vol_t + e_t$$

The *R-square* of the regression remains high at 0.75.

The fact that high exchange rate volatility is associated with asymmetric dependence appears to be consistent with the results in the literature, since asymmetric dependence was mainly found to be present in international equity markets (see Longin and Solnik, 2001). Our own results suggest the presence of asymmetric dependence in international bond markets as well.

Intuitively, the persistence of each dependence regime depends on the persistence of exchange rate volatility. A high exchange rate volatility increases extreme comovements. When bad news in a country combine with a very active currency market, transmission through the latter makes downside joint movements more likely than in a fixed exchange rate regime. This may provide an insight about the strong change in the persistence of different regimes after the introduction of the Euro. Before, the exchange rate between the French Franc and the German Deutsch Mark was especially volatile and this may explain a strong persistence in the asymmetric regime when the model is estimated over this subperiod. After the introduction of the Euro, the volatility is reduced to zero and the normal regime becomes the only persistent regime. By putting the two subperiods together, both regimes appear persistent, which is consistent with our explanation.

These results are consistent with Cappiello et al. (2006) who find a structural break in the dependence structure of European markets after the introduction of the Euro. They find an increase to a near

[29] Since the probability P_t to be in a regime is between 0 and 1, the logistic regression allows us to keep this constraint by proceeding as follows $P_t = \exp(a + Vol_t + \varepsilon_t)/(1 + \exp(a + Vol_t + \varepsilon_t))$ or equivalently $\log(P_t/(1-P_{t))} = a + bVolt + \varepsilon t$ and we can perform the usual regression.

Table 9
Dependence structure tests. Panel A gives results for Dufour (2006) MMC tests of asymmetric dependence. *LR* is the likelihood ratio statistic computed from the data. The *p-value* is obtained from 1000 Monte Carlo repetitions with size 1043 (equal to the sample size) each. Panel B presents Chen and Fan (2005) pseudo-likelihood ratio tests for comparing results from competing multivariate copula models. We compare three models: (i) our regime-switching model with one Gaussian copula regime and one mixture of rotated Gumbel copula regime (N-G RS), (ii) the regime switching with two Gaussian regimes (Norm RS), and (iii) our four-variable mixture of rotated Gumbel copula (Gumbel). For each pair of models the best model at 1% level is indicated in the corresponding cell.

	US–CA		FR–DE
Panel A: Dufour (2006) MMC test			
LR	0.0731		0.7889
p-value	0.0090		0.0000
	N-G RS (Decision at 1% level)	Norm RS	AIC
Panel B: Chen and Fan (2005) test			
US–Canada			
N-G RS			−1.3445
Norm RS	N-G RS		−1.3022
Gumbel	N-G RS	Norm RS	−0.9229
France–Germany			
N-G RS			−4.7217
Norm RS	N-G RS		−3.2309
Gumbel	N-G RS	Gumbel	−4.1763

perfect correlation. However, due to the fact that the dynamic conditional correlation model they use is based on the normal distribution the correlation before the introduction of Euro is misleading since as we find, the dependence between European countries was more asymmetric and therefore the dependence was more in the tail and cannot be completely captured by correlation.

4.4. Empirical implications of the mixture copula constraints

Our mixture copula model aims at capturing extreme dependence, which is of course very useful if an investor wants to manage extreme risks.[30] In Fig. 1, we show that the high exceedance correlation present in the data for the Canada–US pair of equity returns is rather well captured by the model. However, in building the asymmetric copula model to capture tail dependence, we impose some restrictions, either by construction or for parsimony reasons, that may affect the overall level of dependence between pairs of variables. These restrictions are explained in Section 3.3. Chollete et al. (2009) translate these restrictions in terms of Spearman correlation in comparing their canonical vine copulas to our mixture copulas.[31]

4.4.1. Spearman correlation

For two variables X and Y, the Spearman's rho is based on concordance and discordance and is given by the formula

$$\rho_S(X,Y) = 3(\Pr[(X_1 - X_2)(Y_1 - Y_2) > 0] - \Pr[(X_1 - X_2)(Y_1 - Y_2) < 0])$$

where $(X_1 - Y_1)$, $(X_2 - Y_2)$, and $(X_3 - Y_3)$ are three independent random vectors with the same joint distribution like (X, Y), which is defined by the copula C and corresponding marginal distributions. The Spearman rho is related to the copula by the formula[32]

$$\rho_S(X,Y) = \rho_S(C) = 12 \int\int_{I^2} C(u,v)\,du\,dv - 3$$

For the Normal copula, the analytical expression is given by:

$$\rho_S(C_N(\rho)) = (6/\pi)\arcsin(\rho/2),$$

where ρ is the correlation coefficient of a Normal distribution with copula component $C_N(\rho)$. For the Gumbel copula, there is no analytical expression, so we use numerical integration to compute the Spearman rho.

4.4.2. Parametric and nonparametric estimates of the Spearman correlation

Table 10 reports the estimates of the Spearman correlation obtained directly from the data (nonparametric estimate) as well as the estimates obtained from our model with a mixture copula with and without imposing a cross-country constraint of zero dependence between the bond and the equity markets (parametric estimates).[33]

One can say that for the two pairs of countries, Canada–US and France–Germany, the model estimates for the Spearman correlation between the equity markets and between the bond markets are roughly similar to what is observed in the data. For example, for France and Germany, the correlation between the equity markets is 0.84 in the data and 0.83 in the models with and without the constraint. For the correlation between the bond markets, the data estimate is 0.93 but 0.84 and 0.83 with and without the constraint respectively, that is a bit lower. However, where we see a big difference, is in the correlation between the equity and the bond markets, even in the same country. In France it is estimated at 0.49 with the data and at best at 0.10 with the model. This is quite a discrepancy that is due to our model construction. The effect is still present but less damaging in the Canada–US case, where such cross-market correlations are less important, except for the Canadian bond and equity markets. It is estimated at 0.46 in the data and at best at 0.22 in the model. The release of the cross-country constraint helps in getting a bit closer to the data estimates but it does not solve the major problem of underestimating the same country, cross-market correlation. One can conclude that capturing extreme dependence entails some costs and we find that this is specially the case of pairs with lower overall dependence. These costs have to be weighted with the gains of estimating rather well the dependence within markets across countries.

5. Conclusion

We propose a copula-based model of extreme dependence asymmetry that can rationalize the stylized facts put forward by Longin and Solnik. We apply it to the characterization of the extreme dependence in the equity and bond markets of two pairs of countries, the United States and Canada and France and Germany respectively. We capture the well-known strong asymmetric behavior across equity markets, but we also put forward a similar pattern in bond markets. The proposed model allows us to discover a relationship between the filtered probabilities to be in the asymmetric regime and the volatility of exchange rates. This is not possible with the extreme value approach of Longin and Solnik (2001) since only the tails of the distributions are modeled. While useful for extreme risk management, our model has limits for capturing dependence across markets, where the dependence is less strong.

[30] See Tsafack (2009) for the importance of capturing extreme dependence while managing extreme risk.
[31] Chollete et al. (2009) show that the restrictions become increasingly binding when the number of variables increases for mixture copulas. The canonical vine specification generalizes better to higher dimensions but not all applications lend themselves to finding a natural candidate for the first series on which everything is made conditional.
[32] See Nelsen (1999) for more detail about this formulation.
[33] For the results without the constraint, we had to estimate the model again by relaxing the zero dependence between the bond markets and the equity markets in different countries. For space considerations, we do not report the parameter estimates of this larger model. Estimation results are available from the authors upon request.

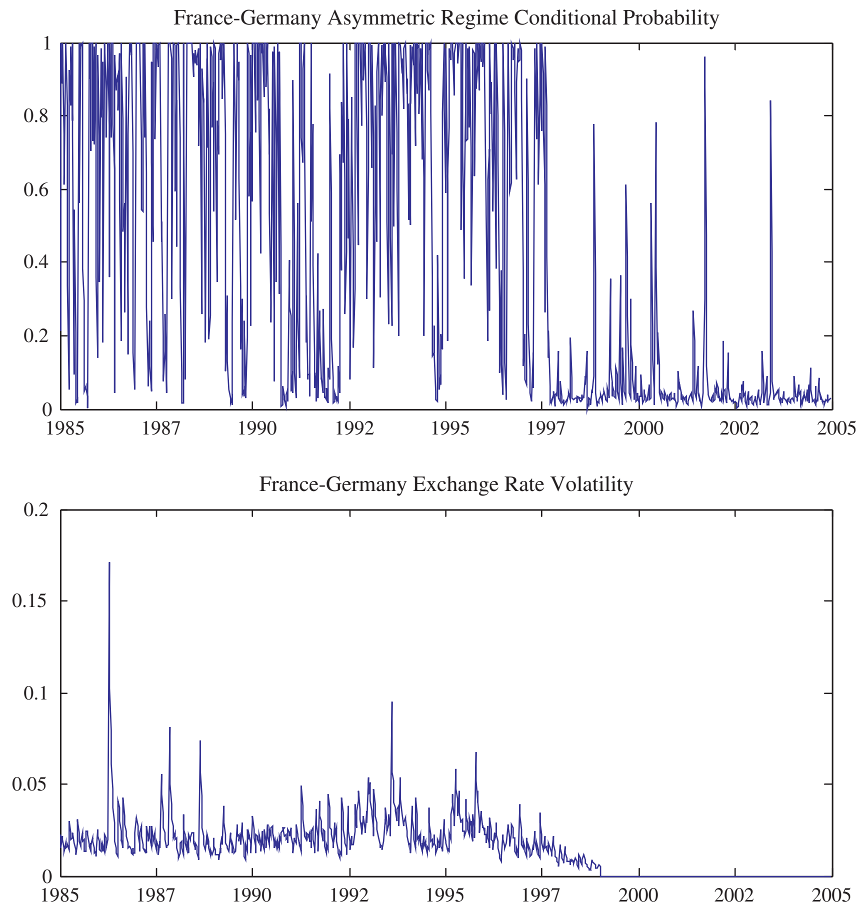

Fig. 5. The upper graph represents the probability to be in the asymmetric regime conditional on available information. The lower graph shows the exchange rate conditional volatility filtered with the M-GARCH (1,1) model.

Table 10
Unconditional Spearman rho between bonds and equity for US, Canada (CA), France (FR), and Germany (DE). Spearman rho from data is the non-parametric estimates using data, while the Spearman rho from models are estimated using ergodic probabilities from the Markov chain combined with the Spearman rho from each regime. The cross-country constraint assumes the independence between equity in one country and bond in the other for asymmetric regime.

	US equity	US bond	CA equity	CA bond	FR equity	FR bond	DE equity
Panel A: Spearman rho from data							
US bond	0.1554						
CA equity	0.6631	0.0775					
CA bond	0.2002	0.4482	0.4590				
FR equity							
FR bond					0.4888		
DE equity					0.8370	0.4515	
DE bond					0.5017	0.9324	0.4879
Panel B: Spearman rho from the model without cross-country constraint							
US bond	0.0286						
CA equity	0.7307	0.0215					
CA bond	0.0982	0.4611	0.2207				
FR bond					0.1025		
DE equity					0.8292	0.0832	
DE bond					0.1280	0.8440	0.0951
Panel C: Spearman rho from the model with cross-country constraint							
US bond	−0.0094						
CA equity	0.7382	−0.0230					
CA bond	0.0350	0.5673	0.1407				
FR bond					0.0640		
DE equity					0.8349	0.0314	
DE bond					0.0540	0.8297	0.0492

Since the exchange rate volatility may be a factor behind the asymmetric behavior of international equity and bond market dependence, it will be interesting to extend the model to incorporate the exchange rate in order to study the portfolio of an international investor. Moreover, the asymmetry put forward between positive and negative extreme returns suggests to investigate the behavior of an investor endowed with disappointment aversion preferences as in Ang et al. (2006).

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Appendix A. Proofs

A.1. Proof of Proposition 2.1

To prove this proposition, we need the two following lemmas:

Lemma 1. *(a) Let $\{f^{(s)}\}_{s=1}^{n}$ be a family of symmetric multivariate density functions of $n(\leqslant\infty)$ variables with same mean. The mixture $f=\sum_{s=1}^{n}\pi_s f^{(s)}$, where $\sum_{s=1}^{n}\pi_s=1$, and $\pi_s\geqslant 0$ for any s, is a symmetric multivariate density function. (b) Moreover for a continuum of symmetric multivariate density function $\{f^{(\sigma)}\}_{\sigma\in A\subseteq\mathbb{R}}$ with same mean, the mixture $f=\int_A \pi_\sigma f^{(\sigma)}d\sigma$, where $\int_A \pi_\sigma d\sigma=1$, is a symmetric multivariate density function.*

Proof. Let μ be the mean of all $f^{(s)}$ (and all $f^{(\sigma)}$)

$$f(\mu-x)=\sum_{s=1}^{n}\pi_s f^{(s)}(\mu-x)$$

by symmetry of all $f^{(s)}$, we have, $\sum_{s=1}^{n}\pi_s f^{(s)}(\mu-x)=\sum_{s=1}^{n}\pi_s f^{(s)}(\mu+x)=f(\mu+x)$ i.e. $f(\mu-x)=f(\mu+x)$ and the part (a) follows. Similarly for mixture of continuum, $f(\mu-x)=\int_A \pi_\sigma f^{(\sigma)}(\mu-x)d\sigma=\int_A \pi_\sigma f^{(\sigma)}(\mu+x)d\sigma=f(\mu+x)$ and we have (b). □

Lemma 2. *Let $\{F^{(s)}\}_{s=1}^{n}$ be a family of bivariate cdf with zero lower (upper) TDC. The mixture $F=\sum_{s=1}^{n}\pi_s F^{(s)}$, where $\sum_{s=1}^{n}\pi_s=1$, and $\pi_s\geqslant 0$, for any s, is a bivariate density function with lower (upper) TDC.*

Proof. we do the proof for lower tail since by "rotation" we have the same result for upper tail.

Let τ_L^F be the lower TDC of F, we have

$$\tau_L^F=\lim_{\alpha\to 0}\Pr\left[X\leqslant F_x^{-1}(\alpha)|Y\leqslant F_y^{-1}(\alpha)\right]$$

$$=\lim_{\alpha\to 0}\frac{\Pr\left[X\leqslant F_x^{-1}(\alpha),Y\leqslant F_y^{-1}(\alpha)\right]}{\Pr\left[Y\leqslant F_y^{-1}(\alpha)\right]}$$

$$=\lim_{\alpha\to 0}\frac{F(F_x^{-1}(\alpha),F_y^{-1}(\alpha))}{F_y(F_y^{-1}(\alpha))}$$

and since $F=\sum_{s=1}^{n}\pi_s F^{(s)}$, we have

$$\tau_L^F=\lim_{\alpha\to 0}\frac{\sum_{s=1}^{n}\pi_s F^{(s)}(F_x^{-1}(\alpha),F_y^{-1}(\alpha))}{\alpha}$$

$$=\lim_{\alpha\to 0}\sum_{s=1}^{n}\pi_s\frac{F^{(s)}(F_x^{-1}(\alpha),F_y^{-1}(\alpha))}{\alpha}$$

$$=\sum_{s=1}^{n}\pi_s\lim_{\alpha\to 0}\frac{F^{(s)}(F_x^{-1}(\alpha),F_y^{-1}(\alpha))}{\alpha}$$

by definition $F^{(s)}\left(F_x^{-1}(\alpha),F_y^{-1}(\alpha)\right)=C^{(s)}\left(F_x^{(s)}\left(F_x^{-1}(\alpha)\right),F_y^{(s)}\left(F_y^{-1}\quad(\alpha)\right)\right)$ where $C^{(s)}$ is the copula and $F_x^{(s)},\ F_y^{(s)}$ the marginal cdf corresponding to $F^{(s)}$, we have

$$\alpha=F_x\left(F_x^{-1}(\alpha)\right)=\sum_{s=1}^{n}\pi_s F_x^{(s)}\left(F_x^{-1}(\alpha)\right)$$

so

$F_x^{(s)}(F_x^{-1}(\alpha))\leqslant\alpha/\pi_s$ for all s and similarly $F_y^{(s)}(F_y^{-1}(\alpha))\leqslant\alpha/\pi_s$, hence

$$\lim_{\alpha\to 0}\frac{F^{(s)}\left(F_x^{-1}(\alpha),F_y^{-1}(\alpha)\right)}{\alpha}=\lim_{\alpha\to 0}\frac{C^{(s)}\left(F_x^{(s)}\left(F_x^{-1}(\alpha)\right),F_y^{(s)}\left(F_y^{-1}(\alpha)\right)\right)}{\alpha}$$

$$\leqslant\lim_{\alpha\to 0}\frac{C^{(s)}(\alpha/\pi_s,\alpha/\pi_s)}{\alpha},\text{ since copula is increasing function}$$

$$=1/\pi_s\lim_{\alpha'\to 0}\frac{C^{(s)}(\alpha',\alpha')}{\alpha'}\text{ by setting }\alpha'=\alpha/\pi_s$$

$$=0,\text{ since }F^{(s)}\text{ and hence }C^{(s)}\text{ is zero lower TDC}$$

we therefore have $\tau_L^F=0$.

The part (i) and (ii) of the proposition is the straightforward application of above lemma

- For GARCH with constant mean and symmetric conditional distribution

  $$X_t=\mu+\Sigma_{t-1}^{1/2}\varepsilon_t$$
  $(+$any GARCH dynamic equation of $\Sigma_{t-1})$

  where ε_t is stationary with symmetric distribution such that $E(\varepsilon_t)=0$. The unconditional distribution of X_t is a mixture of distribution of symmetric variable with same mean μ but possibly different variance covariance matrix. By applying the Lemma 1, we conclude that the unconditional distribution of X_t is symmetric and (i) follows:
- For RS model with zero TDC

  $$X_t=\mu_{s_t}+\Sigma_{s_t}^{1/2}\varepsilon_t$$

  where s_t takes a discrete value. Without loss of generality assume that X_t is bivariate and that $s_t=s$, $\mu+\Sigma^{1/2}\varepsilon_t$ is zero TDC such as in the normal case, therefore the unconditional distribution of X_t is a mixture of distribution with zero TDC. By applying Lemma 2, we conclude that the unconditional distribution of X_t has zero TDC and (ii) follows.

For (iii), with the same notations as Lemma 1, keeping marginal distribution unchanged across mixture components means that. For discrete case

$f^{(s)}(x_1,\ldots,x_n;\delta,\rho) = c^{(s)}(u_1,\ldots,u_n;\theta)\times\prod_{i=1}^n f_i(x_i;\delta_i)$, with $u_i = F_i(x_i;\ \delta_i)$, hence

$$\begin{aligned}f(x_1,\ldots,x_n;\delta,\rho) &= \sum_{s=1}^n \pi_s f^{(s)}(x_1,\ldots,x_n;\delta,\rho)\\ &= \sum_{s=1}^n \pi_s c^{(s)}(u_1,\ldots,u_n;\theta)\times\prod_{i=1}^n f_i(x_i;\delta_i)\\ &= c(u_1,\ldots,u_n;\theta)\times\prod_{i=1}^n f_i(x_i;\delta_i)\end{aligned}$$

with $c(u_1,\ldots,u_n;\theta)=\sum_{s=1}^n \pi_s c^{(s)}(u_1,\ldots,u_n;\theta)$ is the copula of f and we can see that c is a mixture of copula with symmetric TDC and hence is a copula with symmetric TDC. For the continuum case

$$\begin{aligned}f(x_1,\ldots,x_n;\delta,\rho) &= \int_A \pi_\sigma f^{(\sigma)}(x_1,\ldots,x_n;\delta,\rho)d\sigma\\ &= \int_A \pi_\sigma c^{(\sigma)}(u_1,\ldots,u_n;\theta)d\sigma\times\prod_{i=1}^n f_i(x_i;\delta_i)\\ &= c(u_1,\ldots,u_n;\theta)\times\prod_{i=1}^n f_i(x_i;\delta_i)\end{aligned}$$

with $c(u_1,\ldots,u_n;\theta)=\int_A \pi_\sigma c^{(\sigma)}(u_1,\ldots,u_n;\theta)d\sigma$ which is a copula with symmetric TDC for same the reasons mentioned above. □

A.2. Proof of Proposition 3.2

By definition of a copula, we have

$$\eta_t = \begin{bmatrix} f(X_t;\delta,\theta|\underline{X}_{t-1},s_t=1)\\ f(X_t;\delta,\theta|\underline{X}_{t-1},s_t=0)\end{bmatrix} = \begin{bmatrix} c(u_{1,t}(\delta_1),\ldots,u_{4,t}(\delta_4);\theta|s_t=1)\times\prod_{i=1}^4 f_i(x_{i,t};\delta_i)\\ c(u_{1,t}(\delta_1),\ldots,u_{4,t}(\delta_4);\theta|s_t=0)\times\prod_{i=1}^4 f_i(x_{i,t};\delta_i)\end{bmatrix}$$

with $u_{i,t}(\delta_i) = F_i(x_{i,t};\delta_i)$. By denoting $\hat{\xi}_{t|t-1}=(\hat{\xi}^{(1)}_{t|t-1},\hat{\xi}^{(0)}_{t|t-1})'$, the likelihood can be rewritten

$$\begin{aligned}&L(\delta,\theta;\underline{X}_T)\\ &\sum_{t=1}^T \log\left(\hat{\xi}'_{t|t-1}\eta_t\right)\\ &=\sum_{t=1}^T \log\left(\sum_{k=0}^1 \hat{\xi}^{(k)}_{t|t-1} c(u_{1,t}(\delta_1),\ldots,u_{4,t}(\delta_4);\theta|s_t=k)\times\prod_{i=1}^4 f_i(x_{i,t};\delta_i)\right)\\ &=\sum_{t=1}^T \left[\sum_{i=1}^4 \log(f_i(x_{i,t};\delta_i)) + \log\left(\sum_{k=0}^1 \hat{\xi}^{(k)}_{t|t-1} c(u_{1,t}(\delta_1),\ldots,u_{4,t}(\delta_4);\theta|s_t=k)\right)\right]\end{aligned}$$

it follows that

$$L(\delta,\theta;\underline{X}_T) = \sum_{i=1}^4 L_i(\delta_i;\underline{X}_T) + L_C(\delta,\theta;\underline{X}_T)$$

where

$$L_i(\delta_i;\underline{X}_{i,T}) = \sum_{t=1}^T \log f_i\left(x_{i,t};\delta_i|\underline{X}_{i,t-1}\right)$$

$$L_C(\delta,\theta;X) = \sum_{t=1}^T \log\left(\hat{\xi}'_{t|t-1}\eta_{ct}\right)$$

with

$$\eta_{ct} = \begin{bmatrix} c(u_{1,t}(\delta_1),\ldots,u_{n,t}(\delta_n);\theta|s_t=1)\\ c(u_{1,t}(\delta_1),\ldots,u_{n,t}(\delta_n);\theta|s_t=0)\end{bmatrix}$$

by noticing that $\eta_t = \eta_{ct}\times\prod_{i=1}^4 f_i(x_{i,t};\delta_i)$ we have that

$$\hat{\xi}_{t/t} = \left[\hat{\xi}'_{t|t-1}\eta_t\right]^{-1}\left(\hat{\xi}_{t|t-1}\odot\eta_t\right) = \left[\hat{\xi}'_{t|t-1}\eta_{ct}\right]^{-1}\left(\hat{\xi}_{t|t-1}\odot\eta_{ct}\right) \qquad \square$$

Appendix B. Analytical expressions for various copulas

B.1. Normal copula

$$C_N(u_1,\ldots,u_n;\rho) = \Phi_\rho\left(\Phi^{-1}(u_1),\ldots,\Phi^{-1}(u_n)\right)$$

$$C_N(u_1,\ldots,u_n;\rho) = \int_{-\infty}^{\Phi^{-1}(u_1)}\cdots\int_{-\infty}^{\Phi^{-1}(u_n)} \left[(2\pi)^n\det(\rho)\right]^{-\frac{1}{2}}\times \exp\left[-\frac{1}{2}(z'\rho^{-1}z)\right]dz_1,\ldots,dz_n$$

where $z=(z_1,\ldots,z_n)'$, $\rho=(\rho_{ij})^n_{i,j=1}$, with $-\rho_{ij}- \leqslant 1$, $\rho_{ii} = 1$ and ρ positive defined matrix

$$c_N(u_1,\ldots,u_n;\rho) = \left(\det(\rho)\exp[x'\rho^{-1}x - x'x]\right)^{-1/2}$$

with $x = (\Phi^{-1}(u_1),\ldots,\Phi^{-1}(u_n))'$,

Φ is *cdf* of standard normal distribution and Φ_ρ is *cdf* of multivariate normal distribution with correlation matrix ρ.

Tail dependence coefficients are

$$\tau^L = \tau^U = 0$$

B.2. Bivariate Gumbel copula

$$C_G(u,v;\theta) = \exp\left[-\left((-\log(u))^\theta + (-\log(v))^\theta\right)^{1/\theta}\right]$$

$$c_G(u,v;\theta) = \frac{C_G(u,v;\theta)(\log(u).\log(v))^{\theta-1}}{uv\left((-\log(u))^\theta+(-\log(v))^\theta\right)^{2-1/\theta}} \times\left(\left((-\log(u))^\theta+(-\log(v))^\theta\right)^{1/\theta}+\theta-1\right)$$

B.3. Bivariate rotated Gumbel (survival) copula

$$C_{GS}(u,v;\theta) = u+v-1+C_G(1-u,1-v;\theta)$$
$$c_{GS}(u,v;\theta) = c_G(1-u,1-v;\theta)$$

The tail dependence coefficients of C_{GS} are

$$\tau^L = 2-2^{\frac{1}{\theta}} \text{ and } \tau^U = 0$$

so $\theta=\theta(\tau^L)=\frac{\log(2)}{\log(2-\tau^L)}$ and we can re-parameterize the Copula $C_{GS}(u,v;\theta)$ with τ^L as $C_{GS}(u,\ v;\ \tau^L) = C_{GS}(u,\ v;\ \theta(\tau^L))$.

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